Algebra 2A Notes
Unit 10.1 Introduction

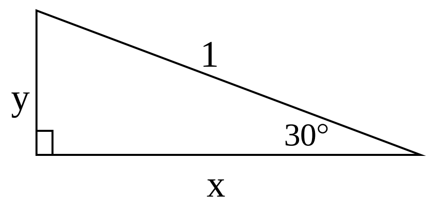
Name ____________________________
Date ________

# Unit Circle

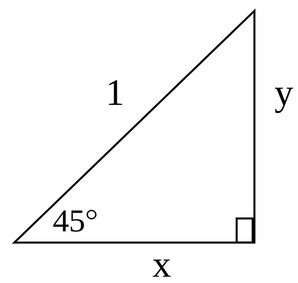
| **Learning Targets** | • <br> • |
|---|---|

Special Right Triangles.

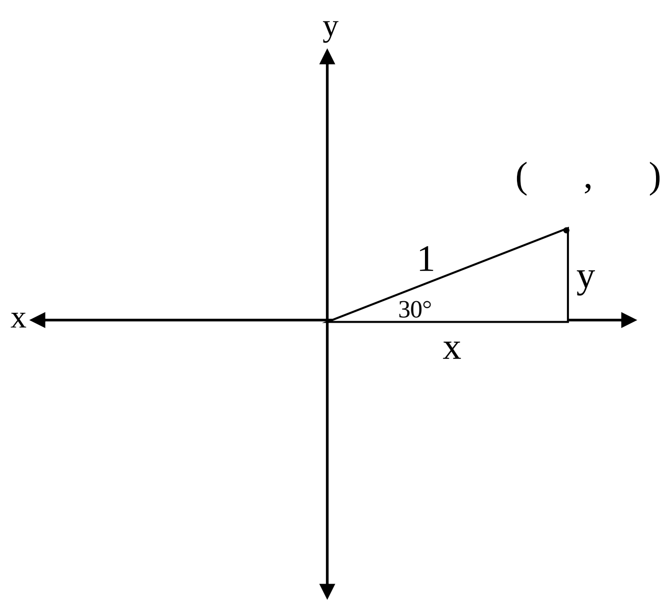
$30° - 60° - 90°$ Triangles:

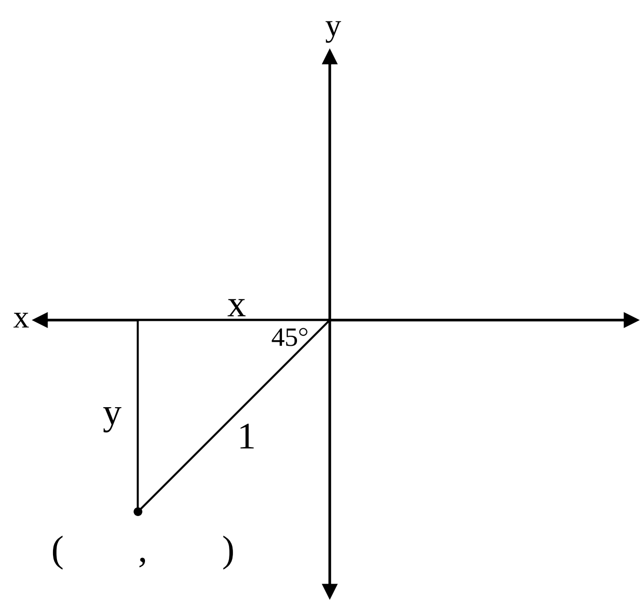



$45° - 45° - 90°$ Triangles:

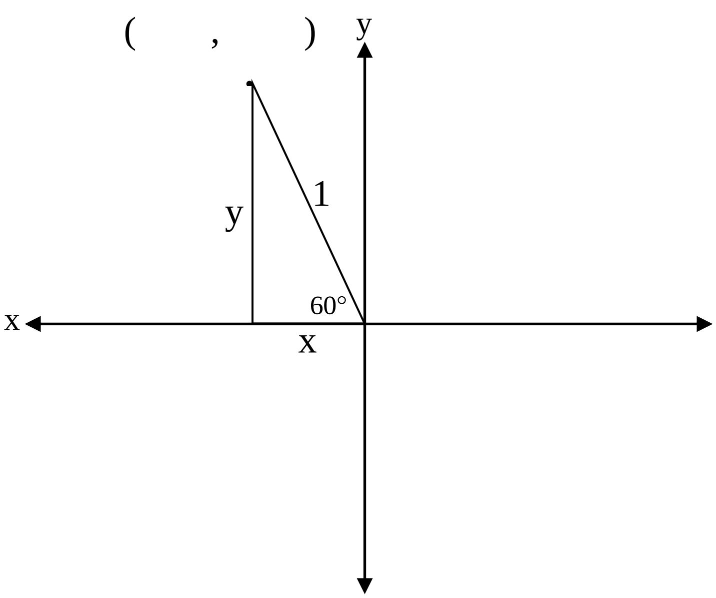


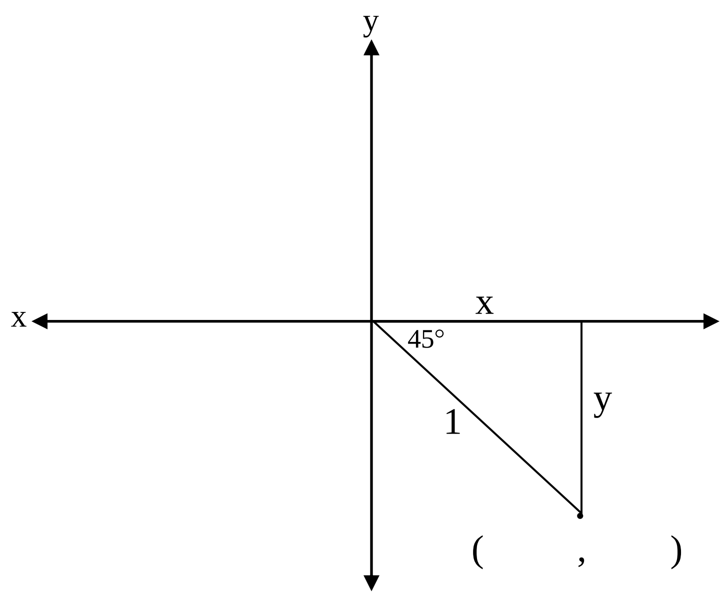

**Example 1:** Find the exact value of x and y on the coordinate plane.



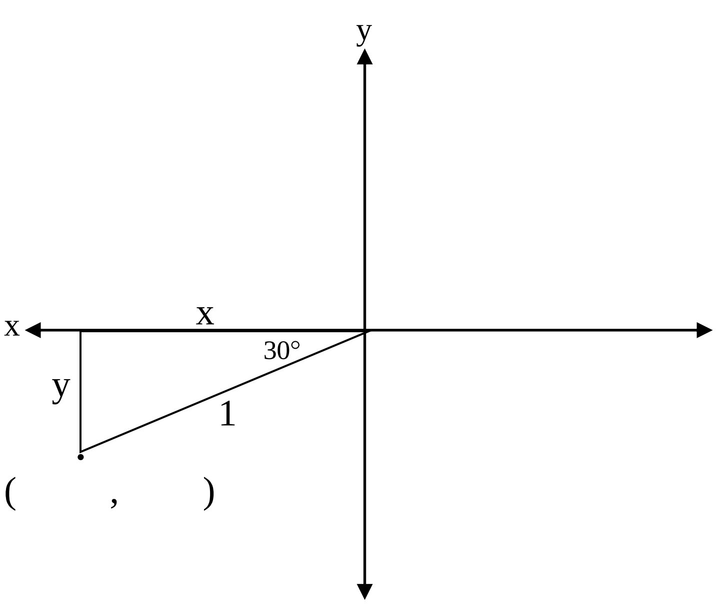



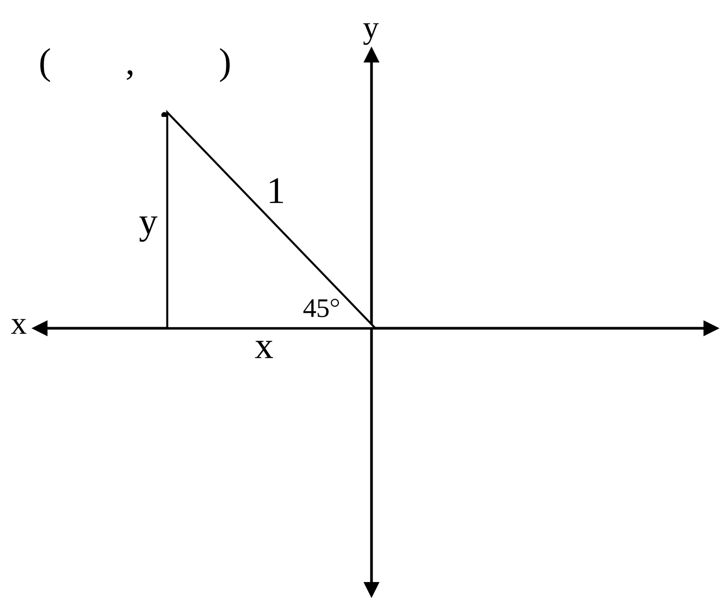
**Example 2:** Find the exact value of x and y on the coordinate plane.



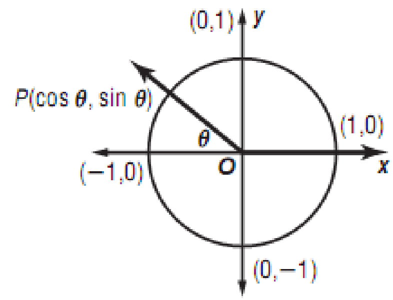

**Your Turn 1:** Find the exact value of x and y on the coordinate plane.



**Example 3:**

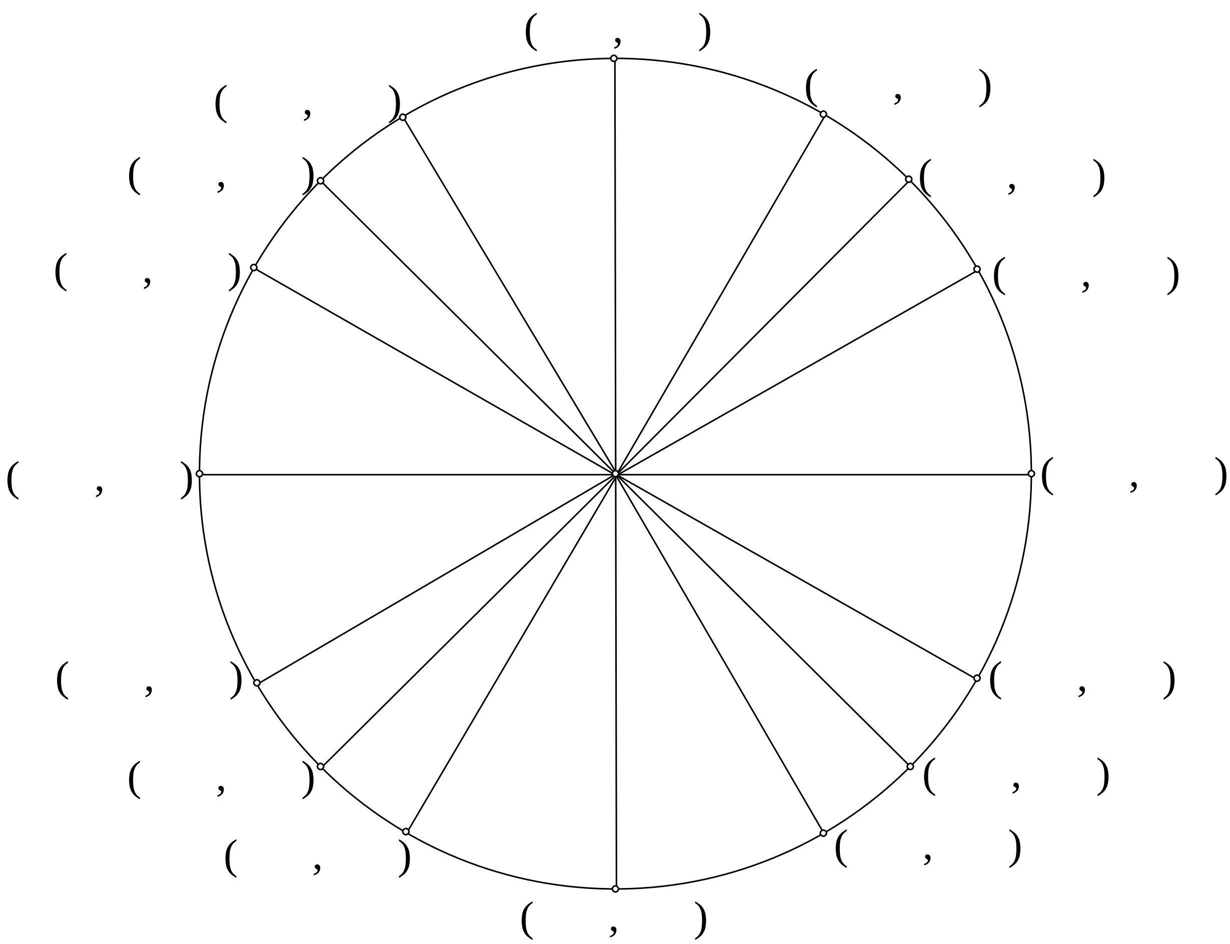
## Unit Circle Definitions

| | | |
|---|---|---|
| **Definition of Sine and Cosine** | If the terminal side of an angle $\theta$ in standard position intersects the unit circle at $P(x, y)$, then $\cos\theta = x$ and $\sin\theta = y$. Therefore, the coordinates of $P$ can be written as $P(\cos\theta, \sin\theta)$. |  |



**Remember:** $P(x, y) = P(\cos\theta, \sin\theta)$ **where** $\cos\theta = x$ **and** $\sin\theta = y$

**Find all the coordinates of the unit circle.** *(Hint: One side of the triangle must be on the x-axis)*

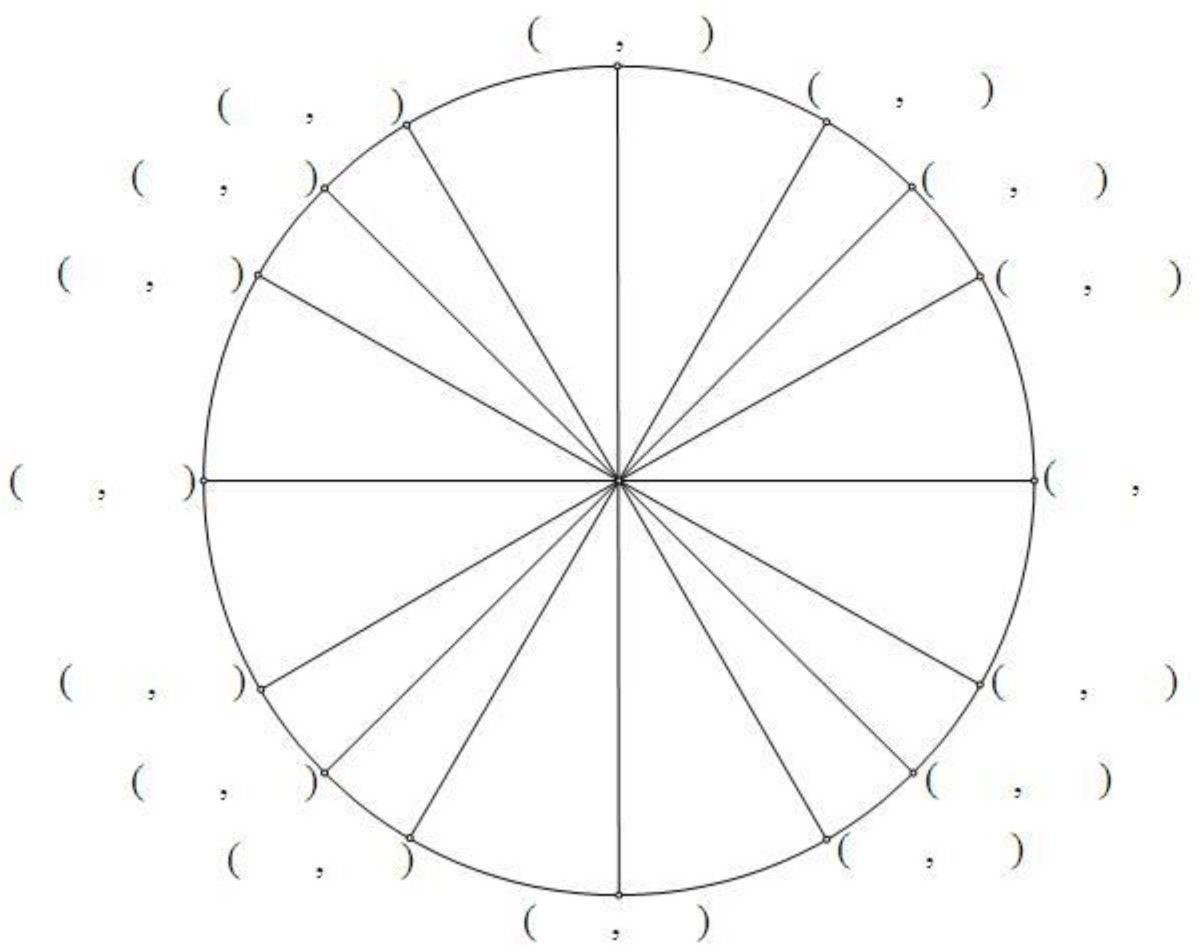


In your own words, what is the **Big Idea** of the lesson.

**Big Idea:** ______________________________________________

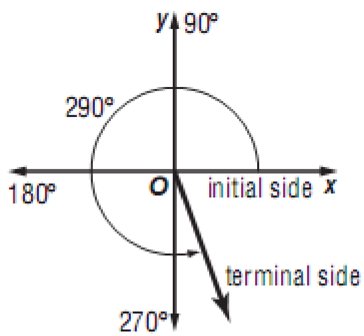
______________________________________________

Radian Measure:  (optional) Divide pi into 1/12's to clarify how reduced fractions are derived.



ACTIVITY: (optional, you need glue, yarn, scissors)

**Angle Measurement:** An angle is determined by two rays. The degree measure of an angle is described by the **amount** and **direction** of rotation from the *initial side* along the positive x-axis to the *terminal side*.




Positive angle:

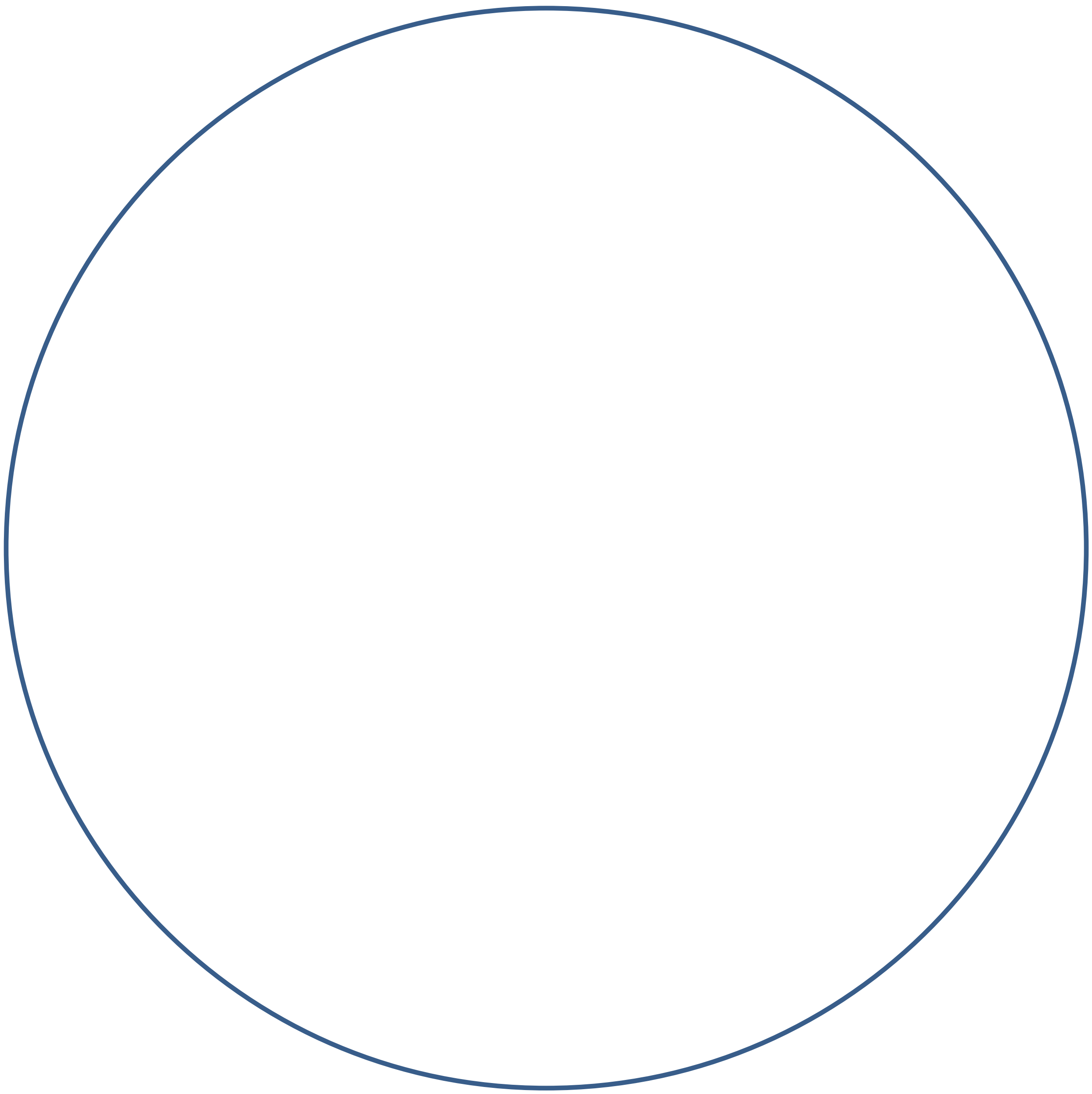
Negative angle:

______ degrees = _______ radians

We will construct a measure equal to 1 radian. Follow these instructions.

1. From the circle below, paper-fold to find the center point. Mark it as *O* for the origin.
2. Using a straight edge, draw a horizontal line through *O*.
3. Mark the horizontal segment at points of intersection with the circle as follows: on the left, *E*; on the right, *A*.
4. Measure a length of yarn from *O* to *A*. This represents the radius of the circle. Be as exact as possible.
5. Lay this piece of yarn on the arc of the circle, starting at point A and marking where the string ends on point *B*. Draw a segment from *O* to *B*. The angle *AOB* measures 1 radian.
6. Cut another length of yarn from *O* to *A*. Lay it on the arc of the circle beginning at *B*, marking where the string ends with a *C*.
7. Cut another length of yarn from *O* to *A*. Lay it on the arc of the circle beginning at *C*, marking where the string ends with a *D*.
8. Your yarn should not "stretch" to meet the point *E*.

Answer these questions:
1. What is the circumference of the circle?
2. What is the circumference of half the circle?
3. How many radians are in the half circle?
4. How many radians are in a full circle?



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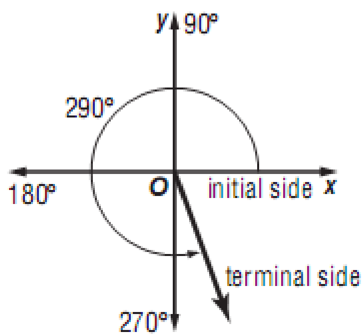
Name ________________________

Date _______

# Angles and Angle Measure

| **Learning Targets** | • <br> • |
|---|---|

**Angle Measurement:** An angle is determined by two rays. The degree measure of an angle is described by the **amount** and **direction** of rotation from the *initial side* along the positive x-axis to the *terminal side*.




Positive angle:

Negative angle:

______ degrees = _______ radians

Example 1: Rewrite each degree measure in radians and each radian measure in degrees.

degrees → radians (multiply by $\frac{\pi}{180°}$)

$90° =$

$180° =$

$270° =$

$360° =$

$45° =$

$60° =$

radians → degrees (multiply by $\frac{180°}{\pi}$)

$-\frac{3\pi}{2} =$

$\frac{5\pi}{3} =$

Your Turn 1: Rewrite each degree measure in radians and each radian measure in degrees.

a. $140°$

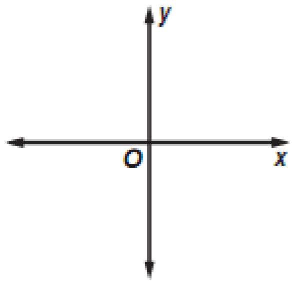
b. $-860°$

c. $-\frac{3\pi}{5}$

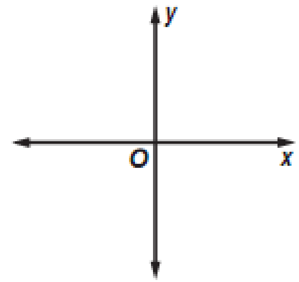
d. $\frac{11\pi}{3}$

**Example 2:** Draw the following angles measures in standard notation.

a. $290°$

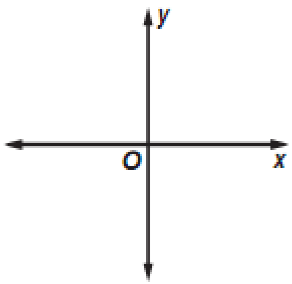



b. $-\dfrac{7\pi}{4}$

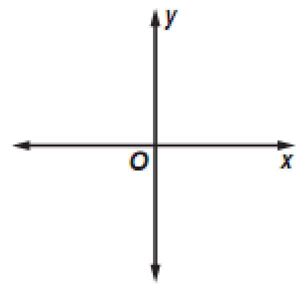



**Your Turn 2:** Draw the following angles measures in standard notation.

a. $-50°$




b. $\dfrac{5\pi}{6}$




**Coterminal Angles:** Two angles in standard position that have the same terminal side. You can find a coterminal angle by **adding or subtracting** _________degrees or _________ radian.

**Example 3:** Find one <u>positive</u> and <u>negative</u> angle measure coterminal with each angle.

a. $690°$

b. $-130°$

c. $\dfrac{3\pi}{8}$

**Your Turn 3:** Find one <u>positive</u> and <u>negative</u> angle measure coterminal with each angle.

a. $-75°$

b. $\dfrac{-13\pi}{6}$

c. $\dfrac{17\pi}{5}$

In your own words, what is the **Big Idea** of the lesson.

**Big Idea:** ________________________________________________________________

________________________________________________________________________

Algebra 2A Notes
Unit 10.3

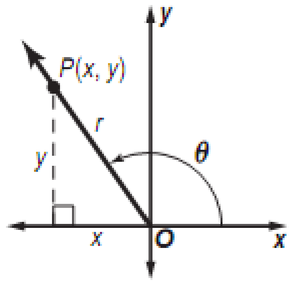
Name ________________________
Date _______

# Trigonometric Functions of General Angles

| **Learning Targets** | • <br> • |
|---|---|

Let $\theta$ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of $\theta$. By the Pythagorean Theorem, the distance $r$ from the origin is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

**Trigonometric Functions, $\theta$ in Standard Position**

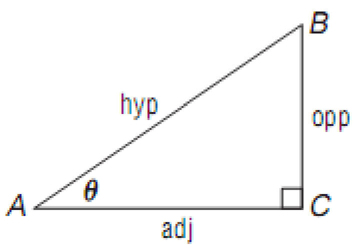



$\sin\theta = \frac{y}{r}$    $\cos\theta = \frac{x}{r}$    $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$    $\sec\theta = \frac{r}{x}$    $\cot\theta = \frac{x}{y}$

If $\theta$ is the measure of an acute angle of a right triangle, *opp* is the measure of the leg opposite $\theta$, *adj* is the measure of the leg adjacent to $\theta$, and *hyp* is the measure of the hypotenuse, then the following are true.

**Trigonometric Functions**




$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

$\csc\theta =$

$\sec\theta =$

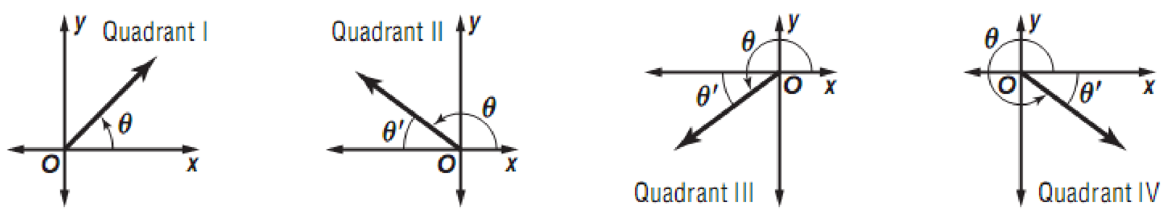
$\cot\theta =$

**Your Turn 1:** Find the exact values of the six trigonometric functions of $\theta$ if the terminal side of $\theta$ contains the following points.

a. $(8, 4)$

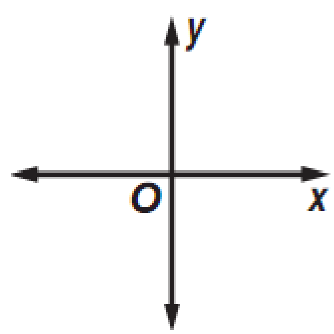
**Reference Angles** If $\theta$ is a nonquadrantal angle in standard position, its reference angle $\theta'$ is defined as the acute angle formed by the terminal side of $\theta$ and the $x$-axis.

**Reference Angle Rule**



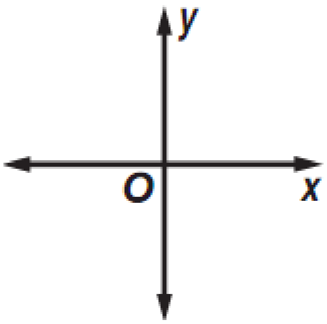

| Quadrant I | Quadrant II | Quadrant III | Quadrant IV |
|---|---|---|---|
| $\theta' = \theta$ | $\theta' = 180° - \theta$ <br> $(\theta' = \pi - \theta)$ | $\theta' = \theta - 180°$ <br> $(\theta' = \theta - \pi)$ | $\theta' = 360° - \theta$ <br> $(\theta' = 2\pi - \theta)$ |

**Example 2:** Sketch an angle of measure $205°$

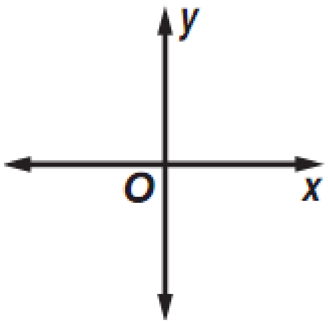


**Your Turn 2: Sketch each angle. Then find its reference angle.**

a. $315°$




b. $-\frac{3\pi}{4}$




**Example 3:** Use a reference angle to find the exact value of the following.

a. $\cos\frac{3\pi}{4}$

b. $\cot 30°$

c. $\csc\frac{11\pi}{4}$

**Your Turn 3: Use a reference angle to find the exact value of the following.**

a. $\tan(-510°)$

e. $\sin(270°)$

b. $\sin\left(-\frac{3\pi}{4}\right)$

f. $\cos(180°)$

c. $\tan(\pi)$

g. $\cot(90°)$

d. $\tan\left(\frac{\pi}{2}\right)$

h. $\sec(2\pi)$

Example 4:

**Suppose $\theta$ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of $\theta$.**

$\sin\theta = \frac{4}{5}$, Quadrant II

Your Turn 4:

**Suppose $\theta$ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of $\theta$.**

$\tan\theta = -\frac{12}{5}$, Quadrant IV

In your own words, what is the **Big Idea** of the lesson.

**Big Idea:** ______________________________________________

______________________________________________

Algebra 2A Notes
Unit 10.4

Name ____________________________
Date ________

# Circular Functions

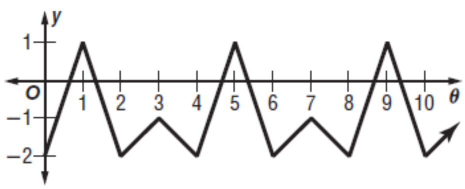
| | |
|---|---|
| **Learning Targets** | • |

## Periodic Functions

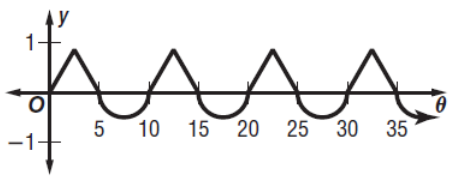
| | |
|---|---|
| **Periodic Functions** | A function is called **periodic** if there is a number *a* such that $f(x) = f(x + a)$ for all $x$ in the domain of the function. The least positive value of *a* for which $f(x) = f(x + a)$ is called the period of the function. |

The sine and cosine functions are periodic; each has a period of 360° or $2\pi$.

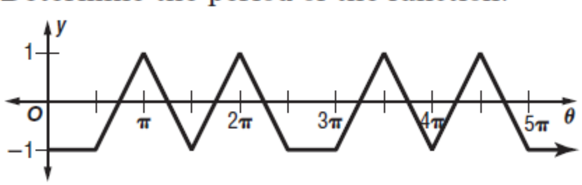
**Example 1:** Determine the period



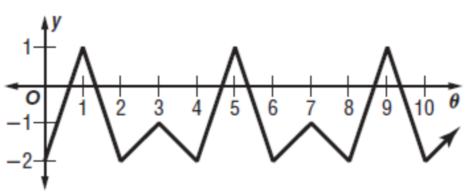

**Example 2** **Determine the period of the function graphed below.**



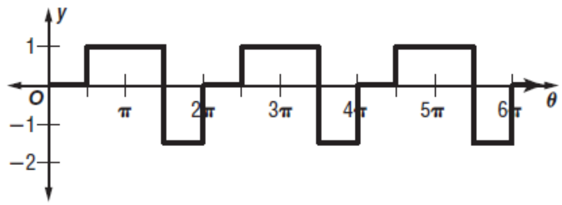

Example 3:
Determine the period of the function.




**Example 4:**



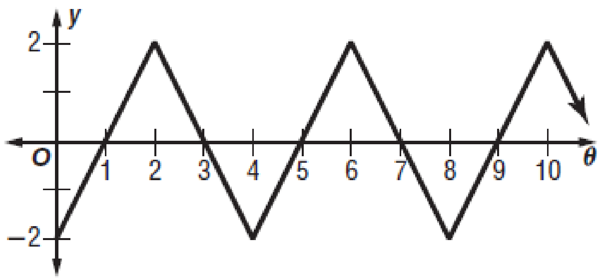
**Example 5:**



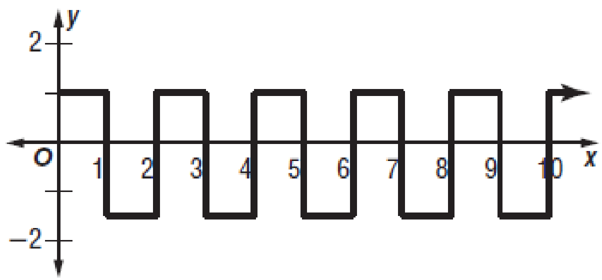

**Your Turn 1:**

**Determine the period of each function.**

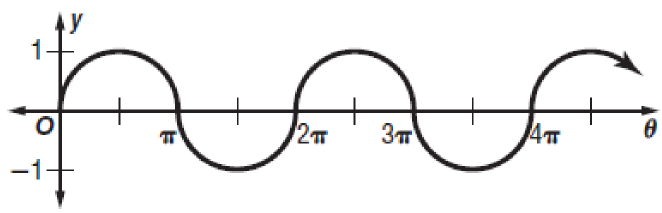
**19.**




**20.**




**21.**




In your own words, what is the **Big Idea** of the lesson.

**Big Idea:** ____________________________________________

____________________________________________

# Practice Test: Trigonometry Part Two

**Find one <u>positive</u> and one <u>negative</u> angle coterminal with each given angle.**

**1) $1260^\circ$ positive: __________ negative: ____________**

**2) $-720^\circ$ positive: __________ negative: ____________**

**3) $\frac{9}{4}\pi$ positive: __________ negative: ____________**

**4) Rewrite each degree measure in radians.**

*4a.* $495^\circ =$

*4b.* $945^\circ =$

*4c.* $-810^\circ =$

*4d.* $-1620^\circ =$

**5) Rewrite each radian measure in degrees.**

*5a.* $-\frac{7\pi}{2} =$

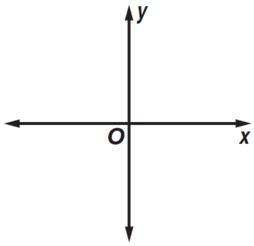
*5b.* $5\pi =$

*5c.* $\frac{15\pi}{4} =$

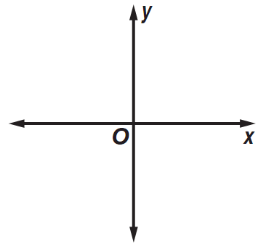
*5d.* $\frac{-9\pi}{2} =$

Find the **exact values** of:

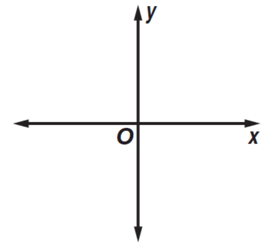
6M. sec $\frac{-\pi}{6}$ = _____



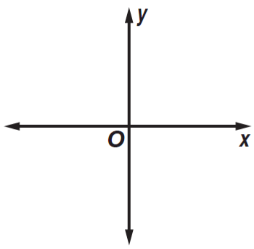

6N. cot -540° = _____



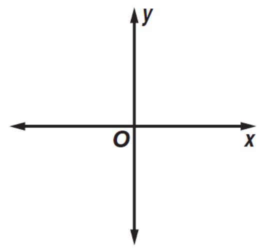

6P. tan 1080° = _____



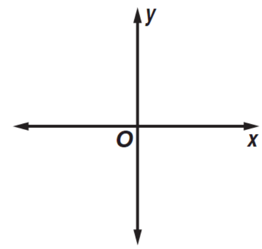

6Q. sin 1080° = _____




6R. csc 1020° = _____




6S. tan 630° =_____




**Find cosine & sine of the angle in standard position (on unit circle) given point P on the terminal side of that angle.**

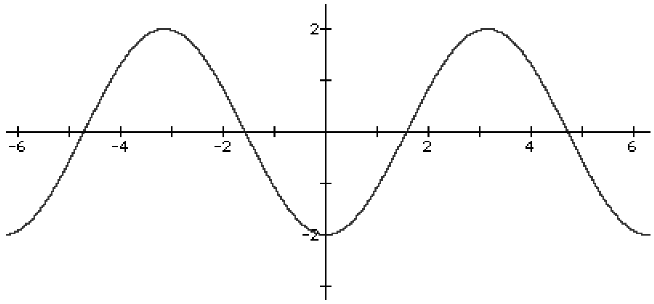
**7.** $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$ $\sin\theta$ = _____ $\cos\theta$ = _____

**8.** $P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\sin\theta$ = _____ $\cos\theta$ = _____

**9.** $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\sin\theta$ = _____ $\cos\theta$ = _____

**10.** $P\left(-\frac{9}{41}, -\frac{40}{41}\right)$ $\sin\theta$ = _____ $\cos\theta$ = _____

11. Use the graph at the right.




a) What **term** is used the describe how high or low a graph changes? ________________

b) What **term** is used to describe how long it takes for the function to repeat?

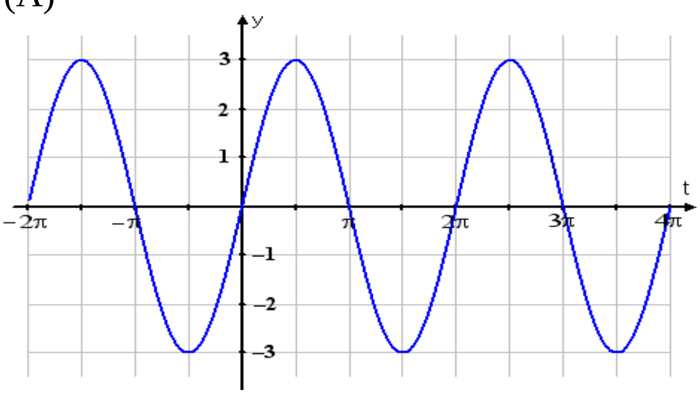
__________________

c) Find the period of the function: ______________

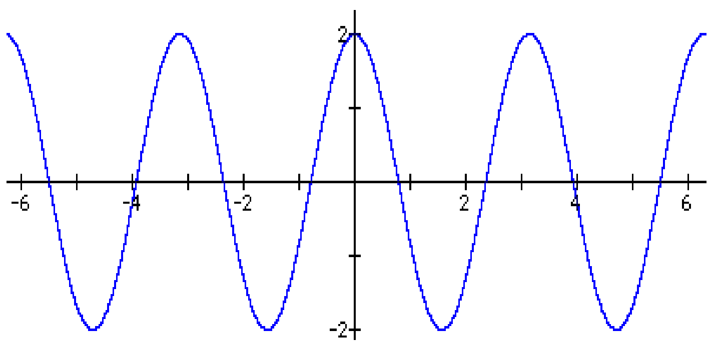
d) Find the amplitude of the function: _____________

12. Use the following two graphs to answer the questions.

(A)




(B)




Period: ___________

Amplitude: ___________

Period: _________

Amplitude: __________

Is graph (A) a **sine function** or **cosine function**? How do you know?

_____________________________________________________________________________

Is graph (B) a **sine function** or **cosine function**? How do you know?

_____________________________________________________________________________