Algebra 2B Notes
Lesson M.1

Name ______________________
Date _______

Introduction to Matrices

Learning Targets	• I can organize data in matrices. • I can solve equations involving matrices.

Organize Data

Matrix	a rectangular array of variables or constants in horizontal rows and vertical columns, usually enclosed in brackets.

$$\textbf{matrix } A \textbf{ if } A = \begin{bmatrix} 13 & 10 & -3 & 45 \\ 2 & 8 & 15 & 80 \end{bmatrix}$$

Element(s):

Dimensions:

Special matrices:

Row matrix:

Column matrix:

Square matrix:

Zero matrix:

Example 1: State the dimensions of each matrix.

1. $\begin{bmatrix} 15 & 5 & 27 & -4 \\ 23 & 6 & 0 & 5 \\ 14 & 70 & 24 & -3 \\ 63 & 3 & 42 & 90 \end{bmatrix}$

2. $\begin{bmatrix} 16 & 12 & 0 \end{bmatrix}$

Your turn 1: State the dimensions of each matrix.

1. $\begin{bmatrix} 71 & 44 \\ 39 & 27 \\ 45 & 16 \\ 92 & 53 \\ 78 & 65 \end{bmatrix}$

2. $\begin{bmatrix} 9 & 3 & -3 & -6 \\ 3 & 4 & -4 & 5 \end{bmatrix}$

Equations Involving Matrices

Equal Matrices	Two matrices are equal if they have the same dimensions and each element of one matrix is equal to the corresponding element of the other matrix.

Example 2: Solve for x and y.

a) $\begin{bmatrix} 3x \\ y \end{bmatrix} = \begin{bmatrix} 28 + 4y \\ -3x - 2 \end{bmatrix}$

b) $\begin{bmatrix} 8x - y & 16x \\ 12 & y - 4x \end{bmatrix} = \begin{bmatrix} 18 & 20 \\ 12 & -13 \end{bmatrix}$

Your Turn 2: Solve for x and y.

a) $\begin{bmatrix} -2y \\ x \end{bmatrix} = \begin{bmatrix} 4 - 5x \\ y + 5 \end{bmatrix}$

b) $\begin{bmatrix} 8x - 6y \\ 12x + 4y \end{bmatrix} = \begin{bmatrix} -3 \\ -11 \end{bmatrix}$

Example 3: Owls' eggs incubate for 30 days and their fledgling period is also 30 days. Swifts' eggs incubate for 20 days and their fledgling period is 44 days. Pigeon eggs incubate for 15 days and their fledgling period is 17 days. Eggs of the king penguin incubate for 53 days, and the fledgling time for a king penguin is 360 days.

Write a 2 x 4 matrix to organize this information. *Source: The Cambridge Factfinder*

Your Turn 3: A travel agent provides for potential travelers the normal high temperatures for the months of January, April, July, and October for various cities. In Boston these figures are $36°, 56°, 82°$ and $63°$. In Dallas they are $54°, 76°, 97°$ and $79°$. In Los Angeles they are $68°, 72°, 84°$ and $79°$. In Seattle they are $46°, 58°, 74°$ and $60°$, and in St. Louis they are $38°, 67°, 89°$ and $69°$.

Organize this information in a 4 x 5 matrix. *Source: The New York Times Almanac.*

In your own words, what is the **Big Idea** of the lesson.

Big Idea: ____

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Algebra 2B Notes
Lesson M.2

Name ______________________________
Date ________

Operations with Matrices

Learning Targets	- I can add matrices. - I can subtract matrices. - I can multiply a matrix by a scalar.

Add and Subtract Matrices

Addition of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} + \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a+j & b+k & c+l \\ d+m & e+n & f+o \\ g+p & h+q & i+r \end{bmatrix}$

Subtraction of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} - \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a-j & b-k & c-l \\ d-m & e-n & f-o \\ g-p & h-q & i-r \end{bmatrix}$

When is it NOT POSSIBLE to add or subtract matrices?

Scalar Multiplication

You can multiply an $m \times n$ matrix by a scalar k.

Scalar Multiplication	$k\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

Example 1:

Find $A + B$ if $A = \begin{bmatrix} 6 & -7 \\ 2 & -12 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ -5 & -6 \end{bmatrix}$

Example 2:

Find $A - B$ **if** $A = \begin{bmatrix} -2 & 8 \\ 3 & -4 \\ 10 & 7 \end{bmatrix}$ **and** $B = \begin{bmatrix} 4 & -3 \\ -2 & 1 \\ -6 & 8 \end{bmatrix}$

Your turn 1: Perform the indicated operation. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 6 & -5 & 9 \\ -3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 2 \\ 6 & 9 & -4 \end{bmatrix} =$

2. $\begin{bmatrix} 8 & 7 \\ -10 & -6 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 2 & -12 \end{bmatrix} =$

3. $\begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 & -2 \end{bmatrix} =$

Example 3: Multiplying by a scalar.

$$3\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

Example 4:

If $A = \begin{bmatrix} 4 & 0 \\ -6 & 3 \end{bmatrix}$ **and** $B = \begin{bmatrix} -1 & 5 \\ 7 & 8 \end{bmatrix}$,

find $3B - 2A$.

Your Turn 2: Perform the indicated operation.

1. $6\begin{bmatrix} 2 & -5 & 3 \\ 0 & 7 & -1 \\ -4 & 6 & 9 \end{bmatrix}$

2. $-\frac{1}{3}\begin{bmatrix} 6 & 15 & 9 \\ 51 & -33 & 24 \\ -18 & 3 & 45 \end{bmatrix}$

3. $3\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix} - 2\begin{bmatrix} -1 & 2 \\ -3 & 5 \end{bmatrix}$

In your own words, what is the **Big Idea** of the lesson.

Big Idea: ____

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Name ________________________

Date _______

Multiplying Matrices

Learning Targets	• I can multiply matrices. • I can use the properties of matrix multiplication.

Multiply Matrices You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

Multiplication of Matrices	$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$

Example 1: Find AB and the dimension of the product.

$$A = \begin{bmatrix} -4 & 3 \\ 2 & -2 \\ 1 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$$

Example 2: Find the product, if possible. If it is not possible, write *undefined*.

a) $\begin{bmatrix} -1 & 0 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$

b) $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & -2 \\ -3 & 1 & 1 \end{bmatrix}$

Your Turn 1: Find the product, if possible. If it is not possible, write *undefined*.

a) $\begin{bmatrix} 3 & -2 \\ 0 & 4 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 6 & 10 \\ -4 & 3 \\ -2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 4 & -3 \end{bmatrix}$

c) $\begin{bmatrix} 7 & -2 \\ 5 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix}$

Multiplicative Properties The Commutative Property of Multiplication does *not* hold for matrices.

Properties of Matrix Multiplication	For any matrices A, B, and C for which the matrix product is defined, and any scalar c, the following properties are true.
Associative Property of Matrix Multiplication	$(AB)C = A(BC)$
Associative Property of Scalar Multiplication	$c(AB) = (cA)B = A(cB)$
Left Distributive Property	$C(A + B) = CA + CB$
Right Distributive Property	$(A + B)C = AC + BC$

Where **c** is a scalar (or a constant). A, B, and **C** is a matrix.

Example 3: Find $(\boldsymbol{A} + \boldsymbol{B})\boldsymbol{C}$

1) $\boldsymbol{A} = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}$, **and** $\boldsymbol{C} = \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$

Example 3 (cont.): Find $AC + BC$

2) $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$

Does $(A + B)C = AC + BC$?

Your Turn 2:

Use $A = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}, B = \begin{bmatrix} 6 & 4 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & -3 \end{bmatrix}$, and scalar $c = -4$

to determine whether each of the following is true for the given matrices.

1) $c(AB) = (cA)B$

Your Turn 2:

Use $A = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 4 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & -3 \end{bmatrix}$, **and scalar** $c = -4$

to determine whether each of the following is true for the given matrices.

2) $C(A + B) = AC + BC$

In your own words, what is the **Big Idea** of the lesson.

Big Idea: ____

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Algebra 2B Notes
Lesson M.4

Name ________________________
Date _______

Transformations with Matrices

Learning Targets	
	• I can use matrices to determine the coordinates of a translated or dilated figure. • I can use matrix multiplication to find the coordinates of a reflected or rotated figure.

Translation	a transformation that moves a figure from one location to another on the coordinate plane

You can use matrix addition and a translation matrix to find the coordinates of the translated figure.

Example 1: Given a triangle with vertices at coordinates (-5,4), (-1,5), and (-3,-1). Translate the figure 6 units to the right and 4 units down.

Vertex matrix: $\triangle ABC. \begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix}$

Translation matrix:

Image:

Dilation	a transformation in which a figure is enlarged or reduced

You can use scalar multiplication to perform dilations.

Example 2: Given a triangle with vertices at coordinates (-5,4), (-1,5), and (-3,-1).Dilate the figure so the perimeter is 2 times the original perimeter.

Vertex matrix: $\triangle ABC. \begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix}$

Dilation:

Image:

Your Turn 1:

For Exercises 1 and 2 use the following information. Quadrilateral *QUAD* with vertices $Q(-1, -3)$, $U(0, 0)$, $A(5, -1)$, and $D(2, -5)$ is translated 3 units to the left and 2 units up.

1. Write the translation matrix. Write the vertex matrix.

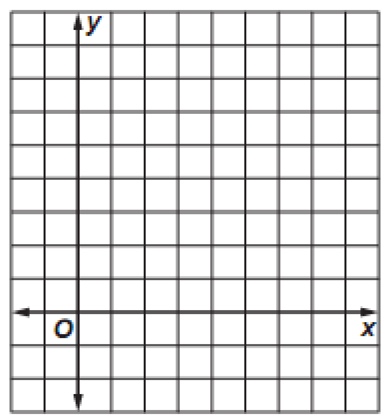
2. Find the coordinates of the vertices of $Q'U'A'D'$.

Example 4:

For Exercises 3–5, use the following information. The vertices of $\triangle ABC$ are $A(4, -2)$, $B(2, 8)$, and $C(8, 2)$. The triangle is dilated so that its perimeter is one-fourth the original perimeter.

3. Write the coordinates of the vertices of $\triangle ABC$ in a vertex matrix.

4. Find the coordinates of the vertices of image $\triangle A'B'C'$.



5. Graph the preimage and the image.

Reflections and Rotations

Reflection Matrices	For a reflection over the:	*x*-axis	*y*-axis	line $y = x$
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Rotation Matrices	For a counterclockwise rotation about the origin of:	90°	180°	270°
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Example 5:

Find the coordinates of the vertices of the image of $\triangle ABC$ with $A(3, 5)$, $B(-2, 4)$, and $C(1, -1)$ after a reflection over the line $y = x$.

Write the ordered pairs as a vertex matrix. Then multiply the vertex matrix by the reflection matrix for $y = x$.

Your Turn 2:

1. The coordinates of the vertices of quadrilateral $ABCD$ are $A(-2, 1)$, $B(-1, 3)$, $C(2, 2)$, and $D(2, -1)$. What are the coordinates of the vertices of the image $A'B'C'D'$ after a reflection over the y-axis?

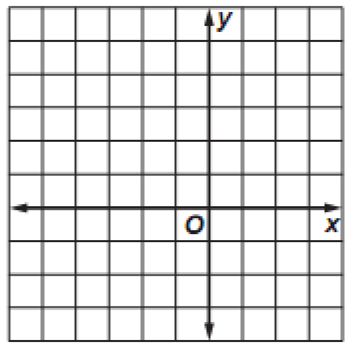
Your Turn 3:

2. Triangle DEF with vertices $D(-2, 5)$, $E(1, 4)$, and $F(0, -1)$ is rotated 90° counterclockwise about the origin.

a. Write the coordinates of the triangle in a vertex matrix.

b. Write the rotation matrix for this situation.

c. Find the coordinates of the vertices of $\triangle D'E'F'$.



d. Graph $\triangle DEF$ and $\triangle D'E'F'$.

In your own words, what is the **Big Idea** of the lesson.

Big Idea: ____

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Algebra 2B Notes
Lesson M.5

Name ________________________
Date _______

Determinants

Learning Targets	• I can evaluate the determinant of a 2 x 2. • I can evaluate the determinant of a 3 x 3.

Determinants of 2 × 2 Matrices

Second-Order Determinant	For the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Example 1: Find the value of the determinant.

$$\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix}$$

Your Turn 1: Find the value of each determinant.

a. $\begin{vmatrix} 11 & -5 \\ 9 & 3 \end{vmatrix}$

b. $\begin{vmatrix} 5 & 12 \\ -7 & -4 \end{vmatrix}$

Determinants of 3 × 3 Matrices

Third-Order Determinants	$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

Example 2: Evaluate the determinant using **EXPANSION BY MINORS**.

$$\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix}$$

Your Turn 2: Evaluate the determinant using **EXPANSION BY MINORS**.

$$\begin{vmatrix} 6 & 1 & 4 \\ -2 & 3 & 0 \\ -1 & 3 & 2 \end{vmatrix}$$

Example 3: Evaluate the determinant using **DIAGONALS.**

$$\begin{vmatrix} 5 & -2 & 2 \\ 3 & 0 & -2 \\ 2 & 4 & -3 \end{vmatrix}$$

Your Turn 3: Evaluate the determinant using **DIAGONALS.**

$$\begin{vmatrix} 4 & 1 & 0 \\ -2 & 3 & 1 \\ 2 & -2 & 5 \end{vmatrix}$$

Area of a Triangle	The area of a triangle having vertices (a, b), (c, d) and (e, f) is $\lvert A \rvert$, where $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$.

Find the area of a triangle with vertices $X(2, -3)$, $Y(7, 4)$, and $Z(-5, 5)$.

In your own words, what is the **Big Idea** of the lesson.

Big Idea: ____

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Algebra 2B Notes
Lesson M.6

Name ____________________
Date _______

Identity and Inverse Matrices

Learning Targets	• I can determine whether two matrices are inverses. • I can find the inverse of a 2 x 2 matrix.

Identity and Inverse Matrices The identity matrix for matrix multiplication is a square matrix with 1s for every element of the main diagonal and zeros elsewhere.

Identity Matrix for Multiplication	If A is an $n \times n$ matrix and I is the identity matrix, then $A \cdot I = A$ and $I \cdot A = A$.

If an $n \times n$ matrix A has an inverse A^{-1}, then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

Identity Matrix:

Example 1: Determine whether X and Y are inverse matrices. Find $X \cdot Y$ and $Y \cdot X$

$$X = \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix}$$

Your Turn 1: Determine whether the following matrices are inverse matrices.
Find $X \cdot Y$ and $Y \cdot X$

$$\begin{bmatrix} 8 & 11 \\ 3 & 4 \end{bmatrix} \text{ and } \begin{bmatrix} -4 & 11 \\ 3 & -8 \end{bmatrix}$$

Find Inverse Matrices

Inverse of a 2 × 2 Matrix	The inverse of a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

If $ad - bc = 0$, the matrix does not have an inverse.

Example 2: **Find the inverse of $N = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}$.**

First find the value of the determinant.

$$N^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} =$$

Your Turn 2: Find the inverse of each matrix, if it exists.

a. $\begin{bmatrix} 24 & 12 \\ 8 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

Example 3: Using inverses to solve a problem.

Decoding a message: Use the alphabet table below and the inverse of the coding matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ to decode this message:

16|30|44|72|27|36|19|38|48|66|51|78

CODE													
A	1	B	2	C	3	D	4	E	5	F	6	G	7
H	8	I	9	J	10	K	11	L	12	M	13	N	14
O	15	P	16	Q	17	R	18	S	19	T	20	U	21
V	22	W	23	X	24	Y	25	Z	26	–	0		

Write the message into a two-column matrix and multiply by the inverse of the coding matrix. (Use the calculator!)

Your Turn 3: Using inverses to solve a problem.

Encoding your own message: Write an appropriate & short message using the alphabet table and write it into a two-column matrix. (Remember #0 represents a space between words.) Multiply your message with the coding matrix B below.

Switch papers with a partner and have the partner decode the message.

Decoding a message: Use the alphabet table below and the inverse of the coding matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ to decode this message:

CODE													
A	1	B	2	C	3	D	4	E	5	F	6	G	7
H	8	I	9	J	10	K	11	L	12	M	13	N	14
O	15	P	16	Q	17	R	18	S	19	T	20	U	21
V	22	W	23	X	24	Y	25	Z	26	–	0		

In your own words, what is the **Big Idea** of the lesson.

Big Idea: ____

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Algebra 2B Notes
Lesson M.7

Name ________________________
Date _______

Using Matrices to Solve Systems of Equations

Learning Targets	• I can write matrix equations to solve systems of equations. • I can solve systems of equations using matrices.

Write Matrix Equations A **matrix equation** for a system of equations consists of the product of the coefficient and variable matrices on the left and the constant matrix on the right of the equals sign.

Example 1: Write a matrix equation for the system of equations.

a. $3x - 7y = 12$
$x + 5y = -8$

b. $2x - y + 3z = -7$
$x + 3y - 4z = 15$
$7x + 2y + z = -28$

Your Turn 1: Write a matrix equation for the system of equations.

a. $2x + y = 8$
$5x - 3y = -12$

b. $a - b + c = 5$
$3a + 2b - c = 0$
$2a + 3b = 8$

Solve Systems of Equations Use inverse matrices to solve systems of equations written as matrix equations.

Solving Matrix Equations	If $AX = B$, then $X = A^{-1}B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.

$$AX = B, \text{ then } X = A^{-1}B$$

Example 2:

Solve $\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

Step 2 Multiply each side of the matrix equation by the inverse matrix.

Your Turn 2: Solve each matrix equation or system of equations by using inverse matrices.

a. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 18 \end{bmatrix}$

b. $3x + 4y = 12$
$5x + 8y = -8$

In your own words, what is the **Big Idea** of the lesson.

Big Idea: ____

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