Lesson 7: Ratios, Proportions, Percentages and Units

Percentages:

Decimals:

Example 1: According to Greg, perfect cherry pies have to 240 cherries to 3 pies. How many cherries does it take to make 9 perfect cherry pies.

$$\frac{240 \text{ cherries}}{3 \text{ pies}} = \frac{x \text{ cherries}}{9 \text{ pies}} \qquad 720 \text{ cherries}$$

Example 2: A pair of jeans that regularly cost $40 is on sale for 30% off. As part of a promotion, each customer will also receive an additional discount on the marked sale price at the cash register. If the final price of the jeans was $22.40, what additional discount was applied to the marked sale price?

A. 12% off
B. 20% off
C. 44% off
D. 56% off

$40 \cdot .7 = \$28$

$\frac{22.40}{28} = .8 \rightarrow$ an additional 20%

Example 3: Today Ebuka opened a new cereal box. He ate a bowl of the cereal, which was 8% of the cereal in the entire box. If Ebuka continues to eat this much cereal in each bowl, approximately how many more bowls of cereal can he expect to get from this box?

A. 7 bowls
B. 8 bowls
C. 11.5 bowls
D. 12.5 bowls

8% of a new box

100% / 8% = 12.5 BUT he already ate 1 bowl so there is only 11.5 bowls left.

Example 4:
Within a certain nursing program, 25% of the class wanted to work with infants, 60% the class wanted to work with the elderly, 10% of the class wanted to assist general practitioners in private practices, and the rest were undecided. What fraction of the class wanted to work with the elderly?

A. ¼
B. 1/10
C. ⅗
D. 1/20

$\frac{60\%}{100\%} = .6$ or $\frac{3}{5}$

Key

Lesson 6: Ratios and Proportions

Example 1: 7 pounds of plums make 8 rolls of fruit leather. If every batch of fruit leather requires the same amount of plums, how many pounds of plums are required to make 20 rolls of fruit leather?

A. 0.875 pounds
B. 2.5 pounds
C. 17.5 pounds
D. 23 pounds

$\frac{7}{8} = \frac{x}{20}$ $\quad \frac{8x}{8} = \frac{140}{8}$

$x = 17.5$

Example 2: A cafeteria with 40 tables can seat 600 people. Some tables can seat 10 people and some can seat 20 people. What is the ratio of the number of 10 person tables to the number of 20 person tables?

A. 1:4
B. 2:1
C. 1:1
D. 4:1

x for # of 10 person tables, y for # of 20 tables

So $x+y=40$ $\quad 10x+20y=600$

$10x$ for the # of people at 10 pt, $20y$ for # of people at 20 pt

$10(x+y=40) \rightarrow 10x+10y=400$

$10x+20y=600$
$-10x+10y=400$

$\frac{10y=200}{10} \quad \frac{}{10}$ $\quad y=20$

Now solve for x.

$x+20=40$
$x=20$

So ratio is 1:1

Example 3: Erika plans to purchase seed to plant grass in a large field. She has a map of the region, and calculates the area of the region on paper to be 180 cm^2. The scale on the map shows that 1 cm = 20 ft. If Erika plans to cover every 400 ft^2 with one pound of seed, approximately how many pounds of seed will she need to cover the entire field?

A. 9 pounds
B. 18 pounds
C. 90 pounds
D. 180 pounds

$(1\text{ cm})^2 = (20\text{ ft})^2$ need it to be cm2

$1\text{ cm}^2 = 400\text{ ft}^2$ $\quad 180\text{ cm}^2 \cdot \left(\frac{400\text{ ft}^2}{1\text{ cm}^2}\right) = 72000\text{ ft}^2$

$72000\text{ ft}^2 \cdot \frac{1\text{lb of seed}}{400\text{ ft}^2} = 180\text{ lbs of seed}$

Example 4: Two cars were traveling 630 miles. Car A traveled an average speed of 70 miles per hour. If Car B traveled 90 miles an hour, how many miles had Car A traveled when Car B arrived at its destination?

A. 490
B. 630
C. 140
D. 700

Car B is going 90 mph

$\frac{630\text{ mi}}{90\text{ mph}} = 7\text{ hours}$

CAR A is going 70 mph

7 hours · 70 mph = 490 miles

Part B:

Example 4:

A car is travelling at x feet per second. The driver sees a red light ahead, and after 1.5 seconds reaction time, the driver applies the brake. After the brake is applied, the car takes $\frac{x}{24}$ seconds to stop, during which time the average speed of the car is $\frac{x}{2}$ feet per second. If the car travels 165 feet from the time the driver saw the red light to the time it comes to a complete stop, which of the following equations can be used to find the value of x?

A) $x^2 + 48x - 3{,}960 = 0$

B) $x^2 + 48x - 7{,}920 = 0$

C) $x^2 + 72x - 3{,}960 = 0$

D) $x^2 + 72x - 7{,}920 = 0$

Strategies we could use:

Break into pieces.

During 1.5 sec reaction time car is still traveling. So goes $1.5x$ ft. Average speed is $x/24$ seconds. Breaking interval is $x/2$ ft/s

Total distance: 165 ft from when he sees red light to stop.

Work/Solution:

$\left(\frac{x}{24}\right)\cdot\left(\frac{x}{2}\right) = \frac{x^2}{48}$ ft

Then $1.5x + \frac{x^2}{48} = 165$ ← total distance. Rewrite $\frac{x^2}{48} + 1.5x - 165 = 0$

multiply each piece by 48 → D

Example 5:

What are the solutions x of $x^3 - 3 = x$?

A) $\frac{-1 \pm \sqrt{11}}{2}$

B) $\frac{-1 \pm \sqrt{13}}{2}$

C) $\frac{1 \pm \sqrt{11}}{2}$

D) $\frac{1 \pm \sqrt{13}}{2}$

$x^3 - x - 3 = 0$

$a = 1 \quad b = -1 \quad c = -3$

Strategies we could use:

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{1^2 - 4(1)(-3)}}{2(1)}$$

Work/Solution

Example 6:

If $x > 0$ and $2x^2 + 3x - 2 = 0$, what is the value of x?

Quadratic Formula $\quad a = 2 \quad b = 3 \quad c = -2$

$$\frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2(2)}$$

can only be positive

So $\frac{2}{4}$ or $\frac{1}{2}$

Example 7:

What is the sum of the solutions of $(2x - 1)^2 = (x + 2)^2$?

$(2x-1)(2x-1) = (x+2)(x+2)$

$4x^2 - 4x + 1 = x^2 + 4x + 4$

$-x^2 - 4x - 4 \quad -x^2 - 4x - 4$

$3x^2 - 8x - 3 = 0$ → Quadratic Formula

$a = 3 \quad b = -8 \quad c = -3$

$$\frac{8 \pm \sqrt{8^2 - 4(3)(-3)}}{2(3)}$$

So $\frac{8+10}{6}$ and $\frac{8-10}{6}$

the sum is $3 - \frac{1}{3} = \frac{8}{3}$

Lesson 9: Statistics and Data

Conditional Probability:

Example 1: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

$$\frac{P(\text{Both})}{P(\text{First test})} = \frac{.25}{.42} = .6 = 60\%$$

Example 2: A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

$$\frac{P(\text{Black or White})}{P(\text{Black})} = \frac{.34}{.47} = .72 = 72\%$$

Mean:

Example 3:

What is the average (arithmetic mean) of the 3 quantities in the list?

12 - n, 12, 12 + n

A. 4
B. 12
C. 18
D. 4 + n/3
E. 12 + n/3

n's cancel

$$\frac{12-n+12+12+n}{3}$$

$$\frac{12+12+12}{3} = 12$$

Example 4:

The average (arithmetic mean) of 6, 19, and x is 19. What is the value of x?

A. 19
B. 25
C. 31
D. 32
E. 57

$$3 \cdot \frac{6+19+x}{3} = 19.3$$

$$25+x = 57$$
$$-25 \quad -25$$

$$x = 32$$