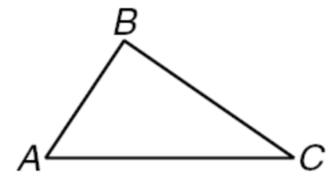
4.1 Angles of Triangles

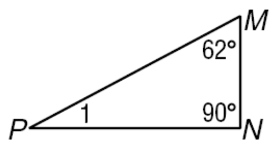
Targets	• I can use the Angle Sum Theorem. • I can use the Exterior Angle Theorem.

Theorem	**Angle Sum Theorem**	• The _______ of the measures of the _________ of a **triangle** is _______.	

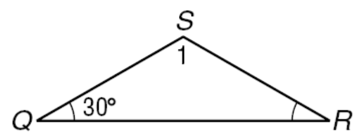
Instruction

Example 1:
Find the measure of each missing angle.

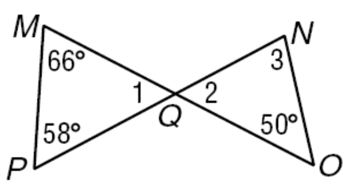
a.



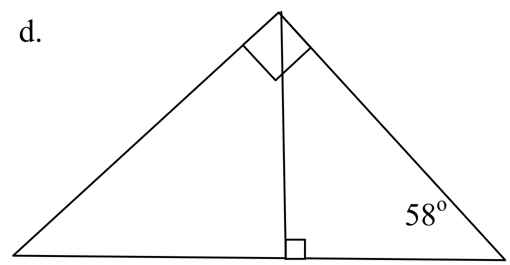
b.

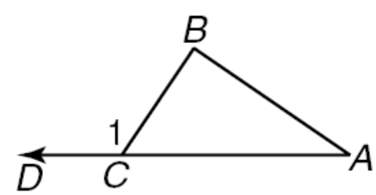
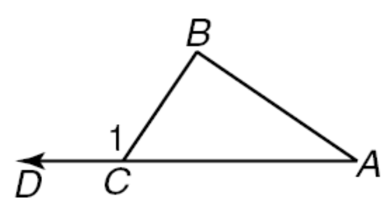


c.



d.



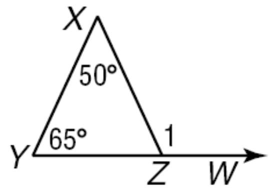
Vocabulary	**Exterior Angle**	• angle formed by one side of a triangle and the extension of another side	
	Remote Interior Angles	• interior angles ______ adjacent to the given exterior angle	
Theorem	**Exterior Angle Theorem**	• The measure of an _______________ __________ of a triangle is equal to the _______ of the measures of the ___ remote interior angles.	

Instruction

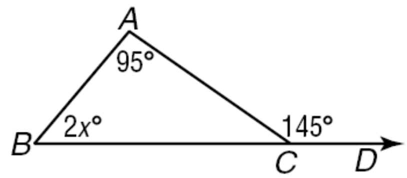
Example 2:

Find the measure of each numbered angle.

a.



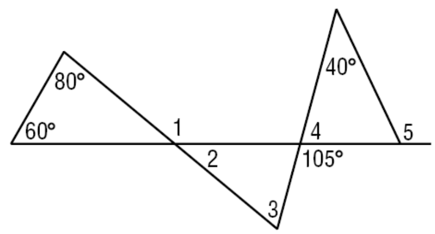
b.



Instruction

Example 3:

Find the measure of each numbered angle.



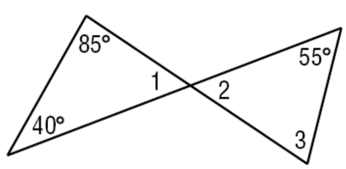
$m\angle 1$ = __________ $m\angle 2$ = __________ $m\angle 3$ = __________

$m\angle 4$ = __________ $m\angle 5$ = __________

Instruction

Your Turn:

Find the measure of each numbered angle.



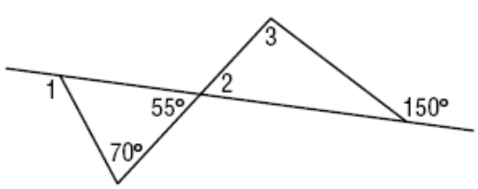
$m\angle 1$ = __________

$m\angle 2$ = __________

$m\angle 3$ = __________

Your Turn:

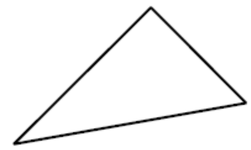
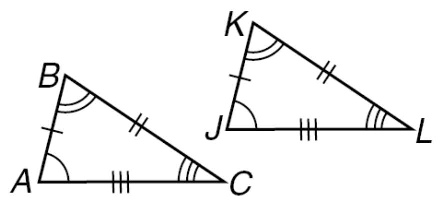
Find the measure of each numbered angle.



$m\angle 1$ = __________ $m\angle 2$ = __________ $m\angle 3$ = __________

4.2 Congruent Triangles

Targets	• I can name and label corresponding parts of congruent triangles.

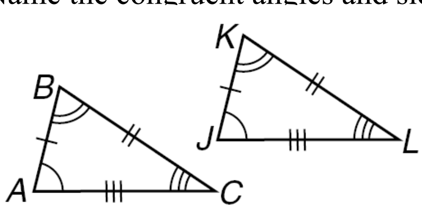
<table>
<tr><td>Instruction</td><td><u>Naming Triangles</u></td><td>• Triangles are named by their ______________.</td><td></td></tr>
<tr><td rowspan="2">Vocabulary</td><td><u>Congruent Triangles</u></td><td>• triangles that are the same _______ and the same ____________
• If all 3 angles and all 3 sides of one triangle are congruent to the corresponding angles and sides in another triangle, then the triangles are congruent.</td><td></td></tr>
<tr><td colspan="3"><u>ABCD Theorem</u>: In congruent figures, the following things are ALWAYS the same (preserved):
A –

B –

C –

D –</td></tr>
</table>

Corresponding Parts of Congruent Triangles are Congruent (CPCTC)

<table>
<tr><td>Instruction</td><td>Example 1:
Name the congruent angles and sides for the pair of congruent triangles below.
</td></tr>
</table>

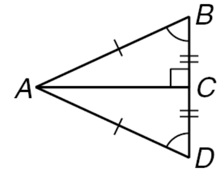
Instruction

Example 2:

Identify the congruent triangles in each example below.
Then name the corresponding congruent angles and congruent sides for the congruent triangles.

a. $\Delta GRD \cong \Delta LXN$

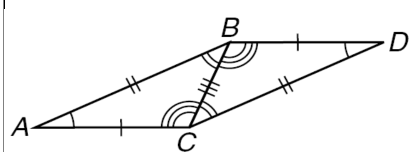
b.



Your Turn

Your Turn:

Name the congruent angles and congruent sides for each pair of congruent triangles.



$\Delta ABC \cong$ ____________

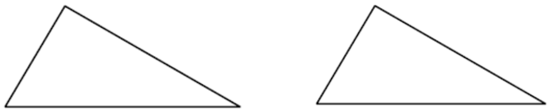
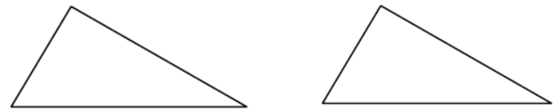
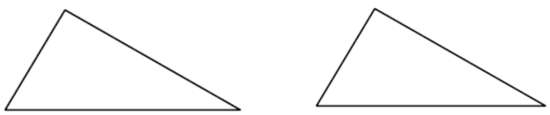
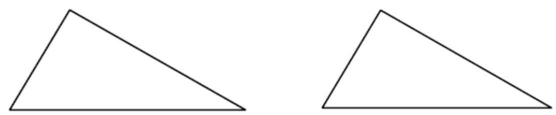
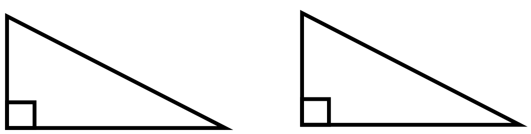
$\angle A \cong$ _________ $\overline{AB} \cong$ _________

$\angle ABC \cong$ _________ $\overline{BC} \cong$ _________

$\angle ACB \cong$ _________ $\overline{AC} \cong$ _________

4.3 Proving Congruence

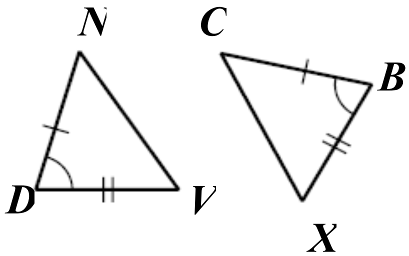
Targets	• I can recognize the SSS, SAS, ASA, AAS and HL Postulates to see if triangles are the same.

Postulate	**Side-Side-Side (SSS) Congruence** 	**Side-Angle-Side (SAS) Congruence** 
Postulate	**Angle-Side-Angle (ASA) Congruence** 	**Angle-Angle-Side (AAS) Congruence** 
Postulate	**Hypotenuse-Leg Congruence (HL)**  ***This is the only case (a right triangle) that SSA is a valid way of proving that two triangles are congruent.	

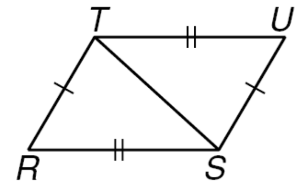
Instruction

Example 1:
Determine which postulate can be used to prove that the given triangles are congruent. Then identify the congruent triangles.

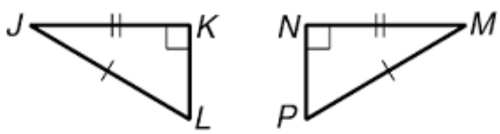
a.



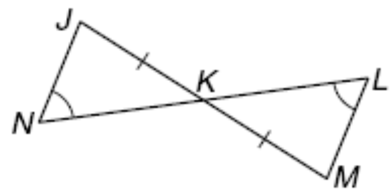
b.



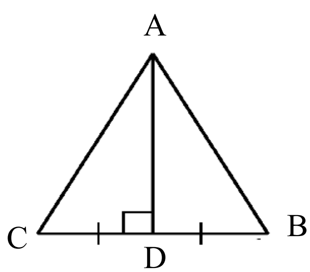
c.



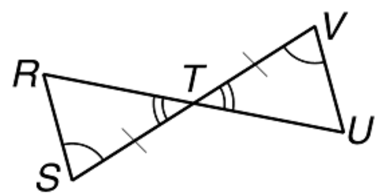
d.



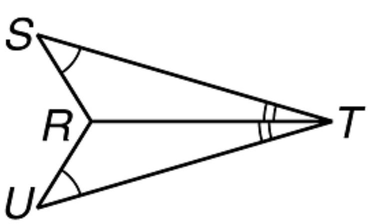
e.



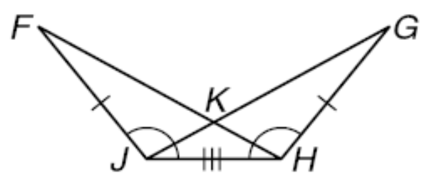
f.



g.



h.



<table>
<tr><td rowspan="1">Instruction</td><td>Example 2:
Draw and Label ΔMWG and ΔARC. Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the <u>ASA</u> Theorem.

$\angle G \cong \angle C$, $\angle M \cong \angle A$</td><td>Example 3:
Draw and Label ΔXYZ and ΔDGK. Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the <u>SAS</u> Theorem.

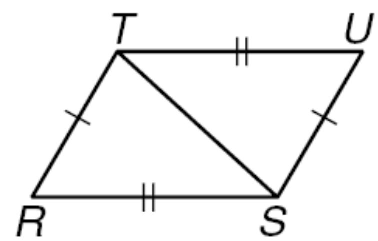
$\overline{XZ} \cong \overline{GK}$, $\angle Z \cong \angle K$</td></tr>
<tr><td>Instruction</td><td colspan="2">Your Turn:
Draw and Label ΔABC and ΔDEF. Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the <u>AAS</u> Theorem.

$\angle A \cong \angle D$, $\overline{BC} \cong \overline{EF}$</td></tr>
</table>

4.4 Proofs with Triangle Congruence

Instruction

Example 1:
Complete the following proof.



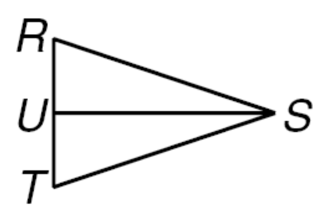
Given: $\overline{RS} \cong \overline{UT}$
$\overline{RT} \cong \overline{US}$

Prove: $\Delta RST \cong \Delta UTS$

Statements	Reasons
1. $\overline{RS} \cong \overline{UT}$	**1.**
2. $\overline{RT} \cong \overline{US}$	**2.**
3.	**3.**
4.	**4.**

Instruction

Example 2:
Complete the following proof.



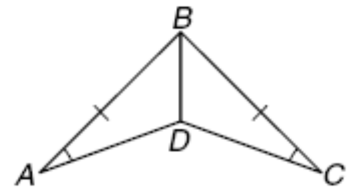
Given: $\overline{RS} \cong \overline{TS}$
$\overline{US}$ bisects $\angle RST$

Prove: $\Delta RSU \cong \Delta TSU$

Statements	Reasons
1. $\overline{RS} \cong \overline{TS}$	**1.**
2. $\overline{US}$ bisects $\angle RST$	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

Instruction

Example 3:
Complete the following proof.



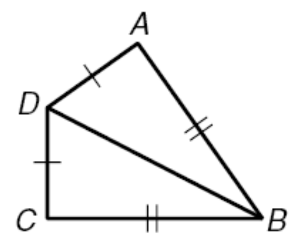
Given: $\overline{AB} \cong \overline{CB}$
$\angle A \cong \angle C$
$\overline{DB}$ bisects $\angle ABC$

Prove: $\Delta ABD \cong \Delta CBD$

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$	**1.**
2. $\angle A \cong \angle C$	**2.**
3. $\overline{DB}$ bisects $\angle ABC$	**3.**
4.	**4.**
5.	**5.**

Instruction

Example 4:
Complete the following proof.



Given: $\overline{AD} \cong \overline{CD}$
$\overline{AB} \cong \overline{CB}$

Prove: $\angle A \cong \angle C$

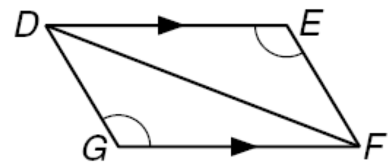
Statements	Reasons
1. $\overline{AD} \cong \overline{CD}$	**1.**
2. $\overline{AB} \cong \overline{CB}$	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

Instruction

Example 5:
Complete the following proof.

Given: $\overline{DE} \parallel \overline{FG}$

$\angle E \cong \angle G$



Prove: $\overline{DG} \cong \overline{FE}$

Statements	Reasons
1. $\overline{DE} \parallel \overline{FG}$	**1.**
2. $\angle E \cong \angle G$	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**

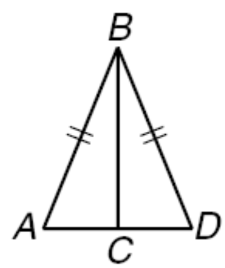
Instruction

Example 6:
Complete the following proof.

Given: $\overline{AB} \cong \overline{DB}$

$\overline{BC} \perp \overline{AD}$

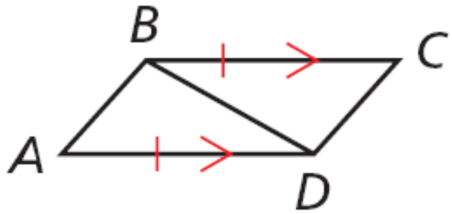
Prove: $\angle A \cong \angle D$



Statements	Reasons
1. $\overline{AB} \cong \overline{DB}$	**1.**
2. $\overline{BC} \perp \overline{AD}$	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

Instruction

Example 7:
Complete the following proof.



Given: $\overline{BC} \parallel \overline{AD}$

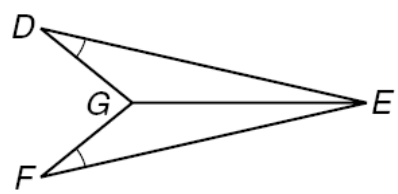
$\overline{BC} \cong \overline{AD}$

Prove: $\Delta ABD \cong \Delta CDB$

Statements	Reasons
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

Instruction

Example 8:
Complete the following proof.



Given: $\angle D \cong \angle F$

$\overline{GE}$ bisects $\angle DEF$

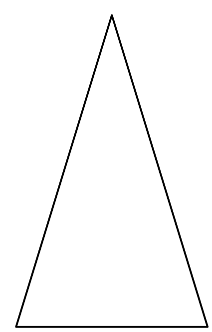
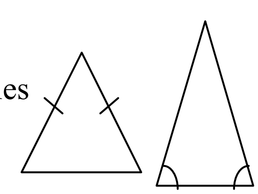
Prove: $\overline{DE} \cong \overline{FE}$

Statements	Reasons
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**

4.5 Isosceles and Equilateral Triangles

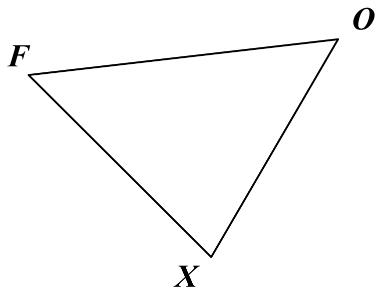
Targets	• I can recognize and use properties of isosceles triangles. • I can recognize and use properties of equilateral triangles.

	Type of Triangle	Definition	Picture
Vocabulary Review	**Isosceles Triangle**	**Triangles with at least two congruent sides**	
	Equilateral Triangle	**Triangles with three congruent sides**	

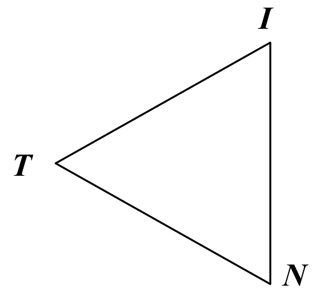
	Isosceles Triangles		
Vocabulary	**Type of Angle**	**Definition**	**Picture**
	Vertex Angle	The angle formed by the ____________________ ____________________	
	Base Angles	The angles ____________ of the ______________ _________	
Theorem	**Isosceles Triangle Theorem** _____________ of a triangle are _______________ if and only if the angles _________________ those sides are ____________________. 		

Example 1:

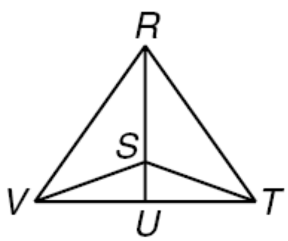
a. If $\angle F \cong \angle O$, name 2 congruent sides.



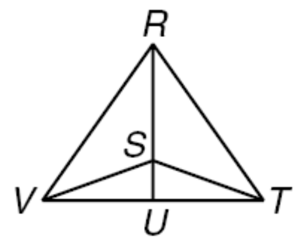
b. If $\overline{TI} \cong \overline{TN}$, name 2 congruent angles.



c. If $\overline{RS} \cong \overline{ST}$, name 2 congruent angles.



d. If $\angle SVU \cong \angle STU$, name 2 congruent segments.

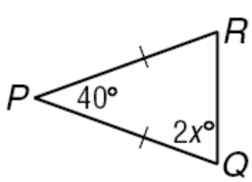


Instruction

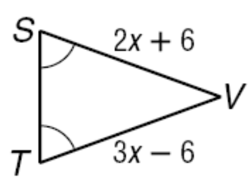
Example 2:

Find the value of x for each of the following figures.

a.



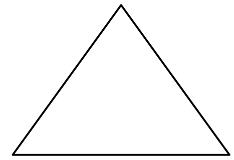
b.



c. ΔLMN is isosceles, $\angle L$ is the vertex angle, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.

Properties

<u>Properties of Equilateral Triangles</u>



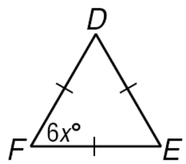
1. An equilateral triangle is ______________________. (all angles are congruent)

2. The measure of each angle of an equilateral triangle is ________.

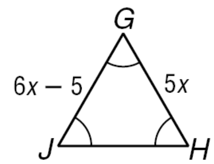
Instruction

Example 3:
Find the value of x for each of the following figures.

a.



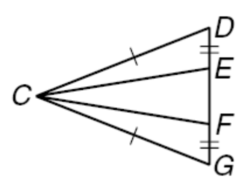
b.



c. ΔFGH is equilateral with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.

Instruction

Example 4:
Complete the following proof.



Given: $\overline{CD} \cong \overline{CG}$ and $\overline{DE} \cong \overline{GF}$

Prove: $\overline{CE} \cong \overline{CF}$

Statements	**Reasons**
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

4.6 Constructing Triangles

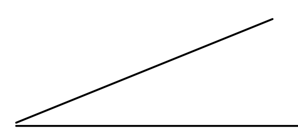
Targets	• I can construct a triangle given 3 side lengths. • I can construct a triangle given 2 side lengths and the included angle. • I can construct a triangle given 2 angles and the included side. • I can construct an equilateral triangle given a side length. • I can construct an isosceles triangle given the base and leg length.

- **Constructing Triangles Given 3 Side Lengths**
  Side-Side-Side (SSS)

Example 1: Construct a triangle that has the following 3 side lengths.

- **Constructing Triangles Given 2 Side Lengths and the Included Angle**
  Side-Angle-Side (SAS)

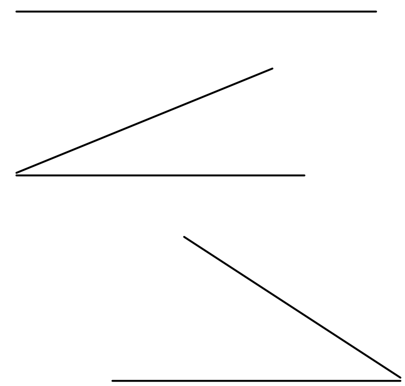
Example 2: Construct a triangle that has the following 2 side lengths and included angle.



- **Constructing Triangles Given 2 Angles and the Included Side**

Angle-Side-Angle (ASA)

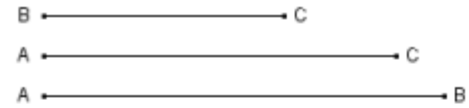
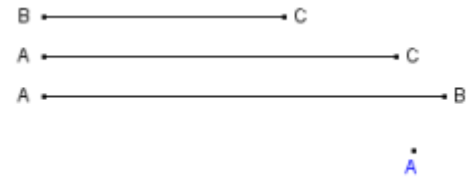
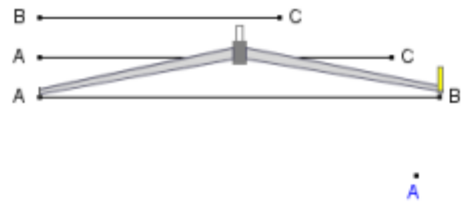
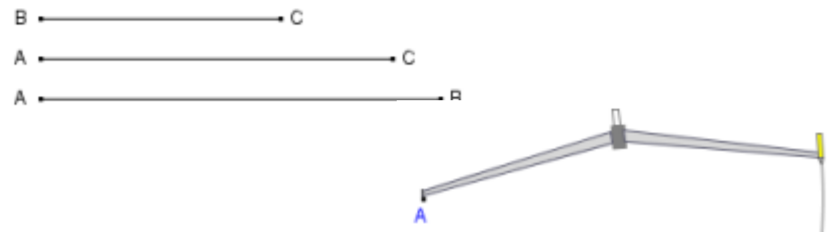
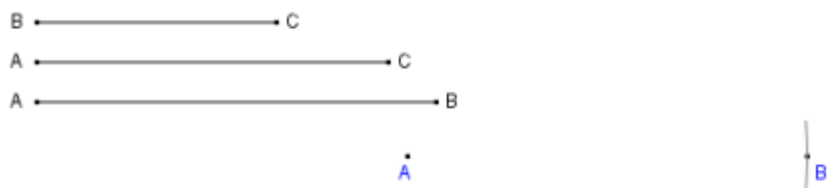
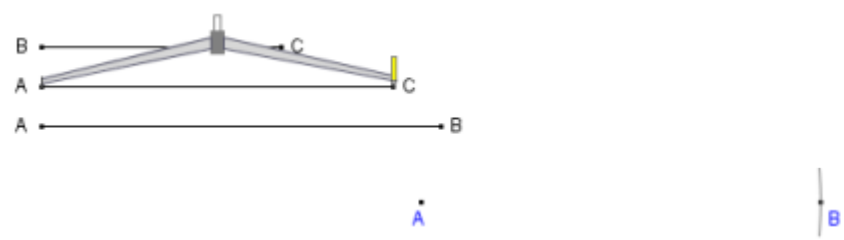
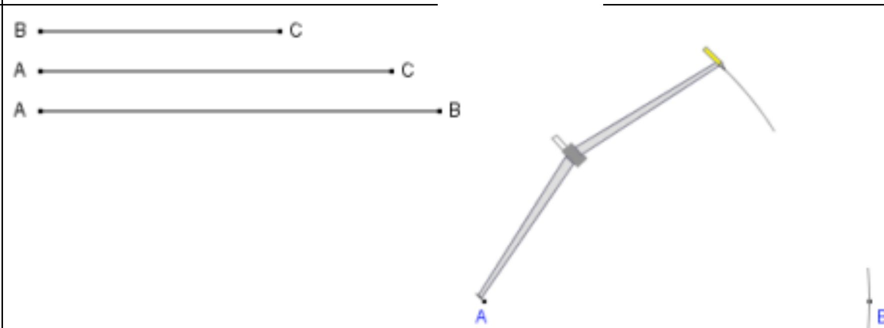
Example 3: Construct a triangle that has the following angles and included side length.

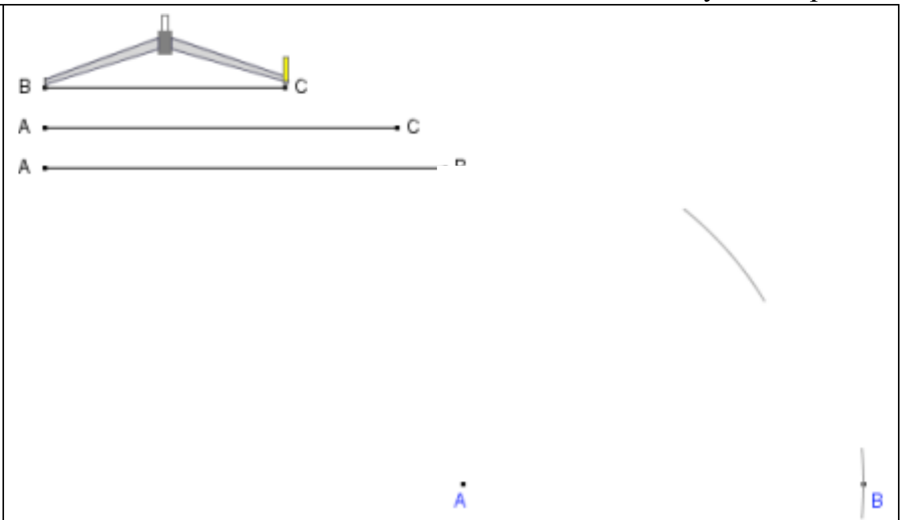
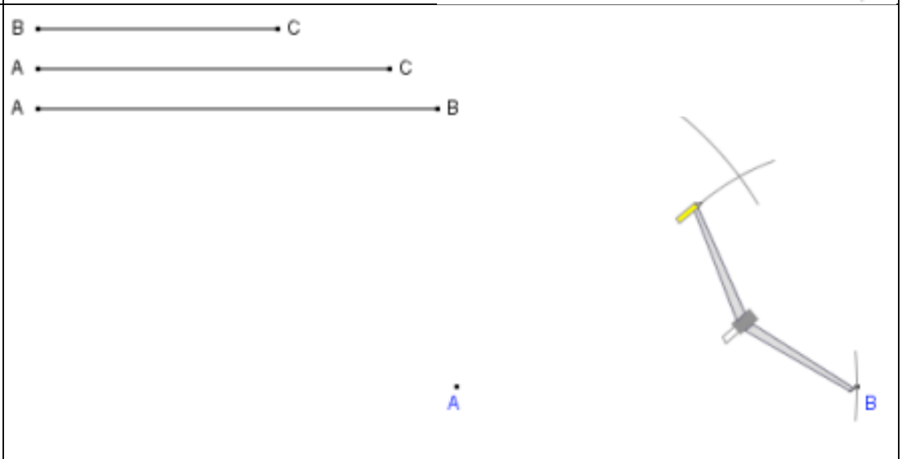
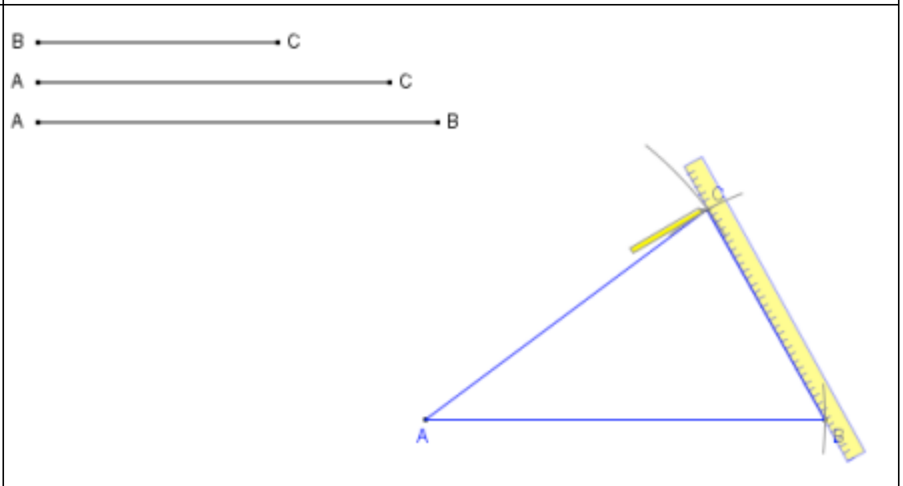
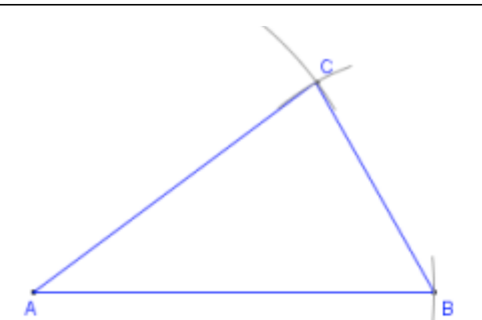


- **Constructing an Equilateral Triangle**

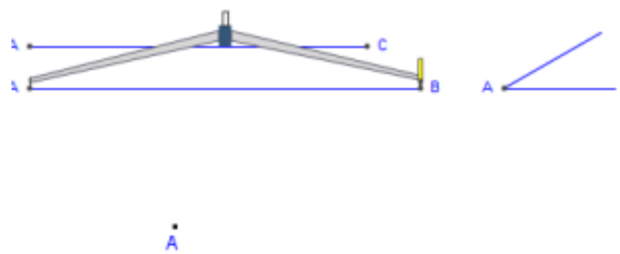
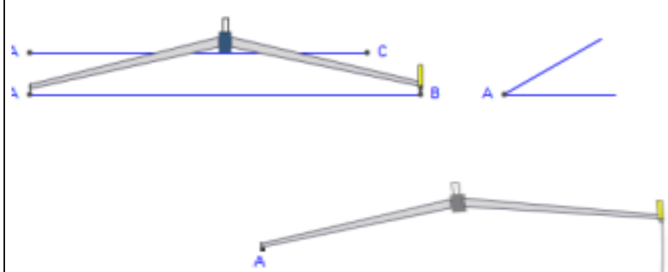
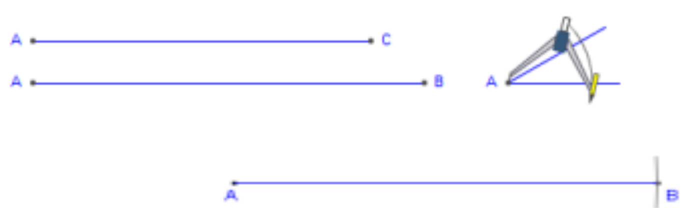
Example 4: Construct an equilateral triangle that has the following side length.

Example 1: CONSTRUCTING A TRIANGLE GIVEN 3 SIDE LENGTHS (SSS)

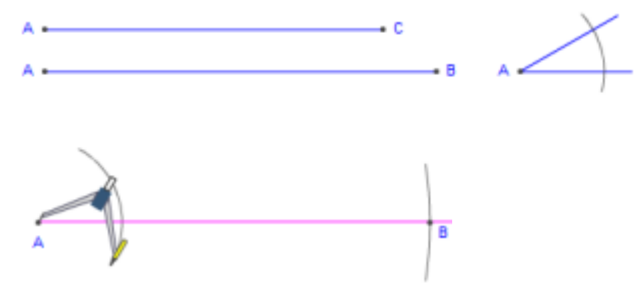
After doing this	Your work should look like this
Start with three line segments that will be the three sides of ΔABC.	
1. Mark a point *A* that will be one vertex of the new triangle.	
2. Set the compass width to the length of the segment *AB*. This will become the base of the new triangle.	
3. With the compass point on *A*, make an arc near the future vertex *B* of the triangle.	
4. Mark a point *B* on this arc. This will become the next vertex of the new triangle.	
5. Set the compass width to the length of the line segment *AC*.	
6. Place the compass point on *A* and make an arc in the vicinity of where the third vertex of the triangle (*C*) will be. All points along this arc are the distance *AC* from *A*, but we do not yet quite know exactly where vertex *C* will be.	

7. Use the compass to measure the length of the segment *BC*, the length of the third side of the triangle.	
8. From point *B*, draw an arc crossing the first. Where these intersect is the vertex *C* of the triangle.	
9. Finally, draw the three sides $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ of the new triangle.	
10. Done!	

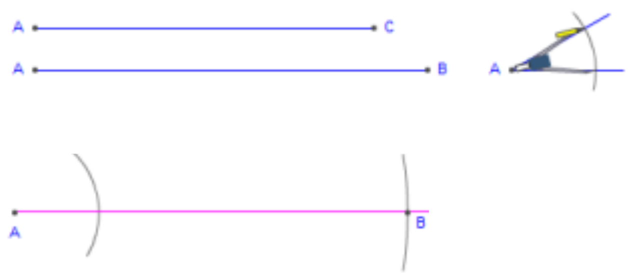
Example 2: **CONSTRUCTING A TRIANGLE GIVEN 2 SIDE LENGTHS and the INCLUDED ANGLE (SAS)**

After doing this	Your work should look like this
Start with two line segments and the included angle.	
1. Mark a point *A* that will be one vertex of the new triangle.	
3. Set the compass width to the length of the given side $\overline{AB}$.	
2. With the compass point on *A*, make an arc near the future vertex *B* of the triangle.	
4. Mark a point *B* on this arc. Then draw the line AB. This will be one side of the new triangle.	
5. With the compass set to any width, from the point *A* on the given angle, draw an arc across both sides of the angle.	

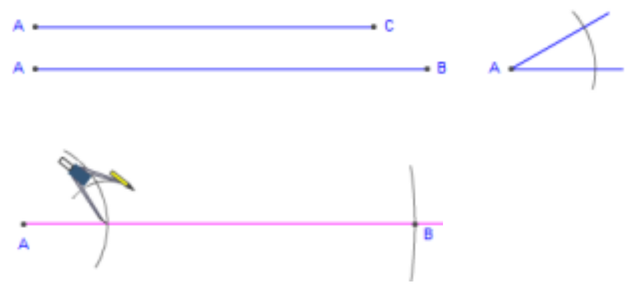
6. Without changing the compass width, draw a similar sized arc at point *A* on the new triangle.



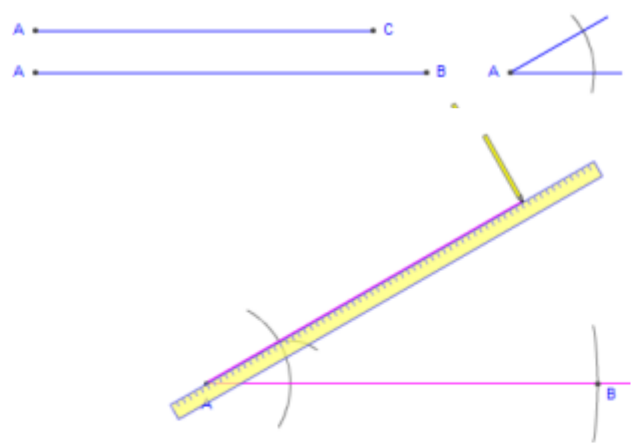
7. Set the compass to the arc width at the given angle A. This the distance between the points where the arc intersects the sides of the angle.



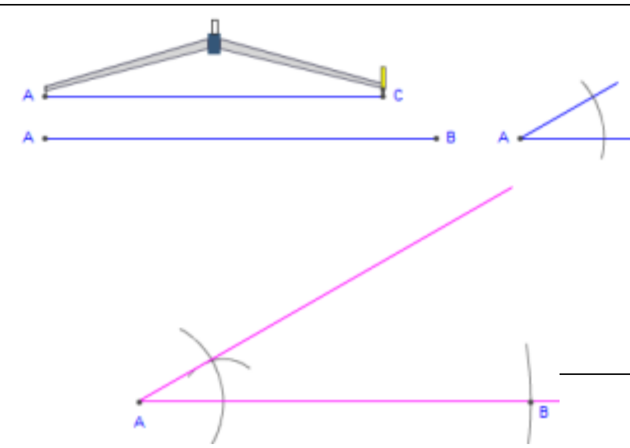
8. Make a similar arc on the new triangle so it crosses the previous arc.

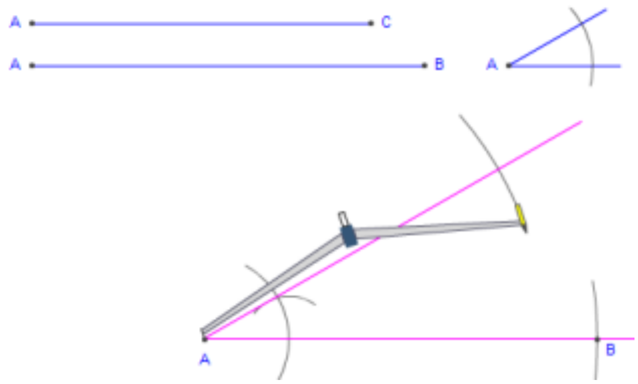
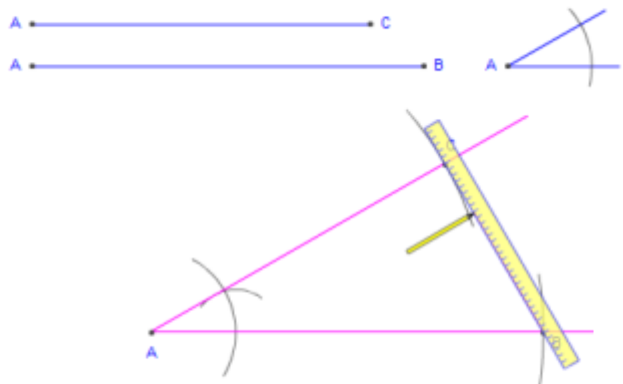
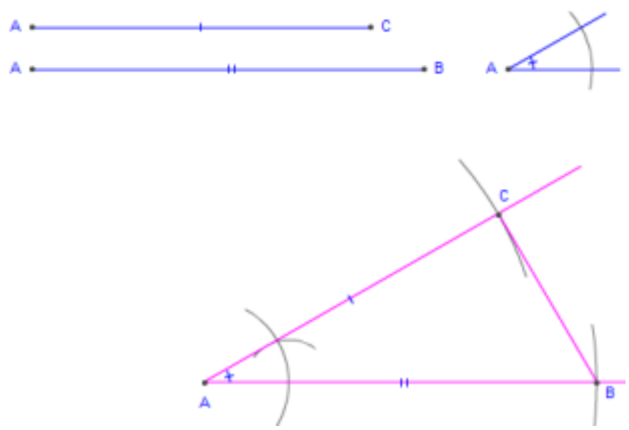


9. Draw a ray from *A*, through where the arcs intersect and onwards. This will become side $\overline{AC}$ of the triangle so make it longer than $\overline{AC}$.

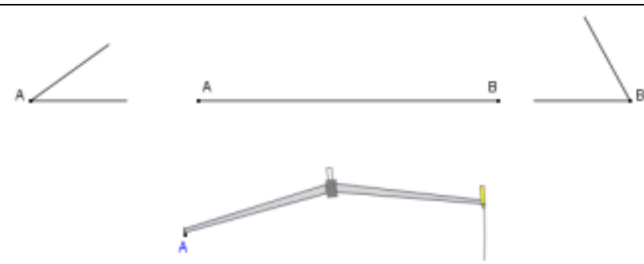
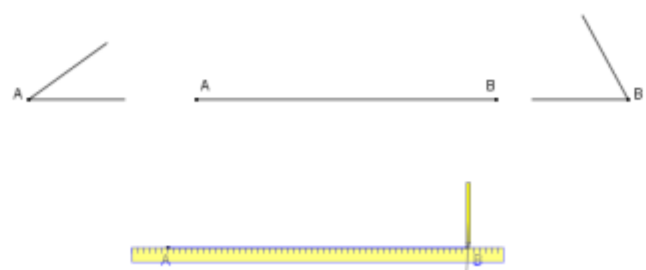
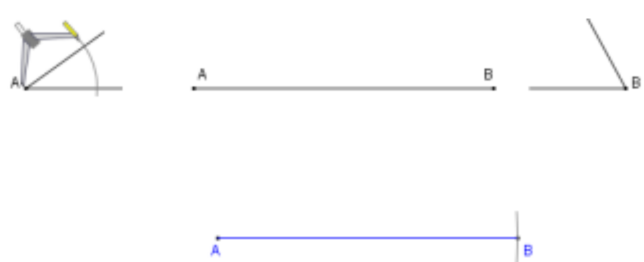


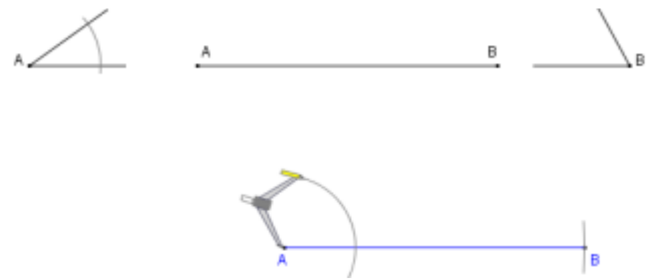
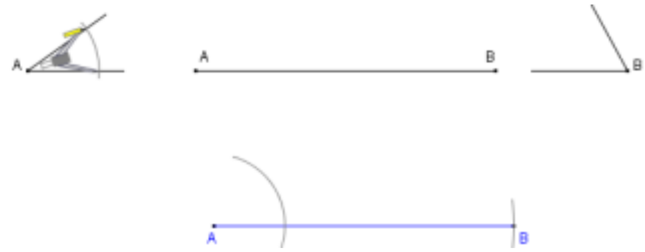
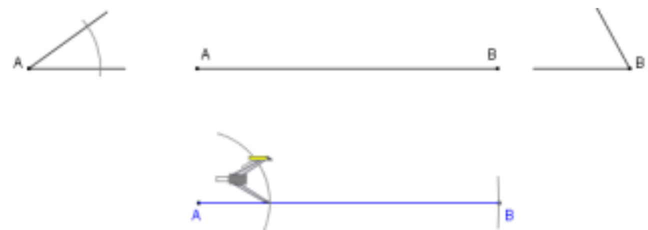
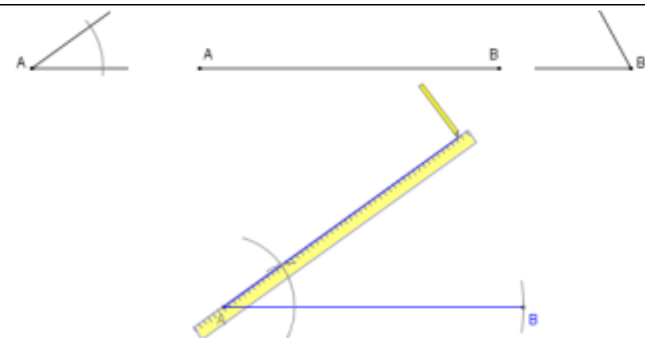
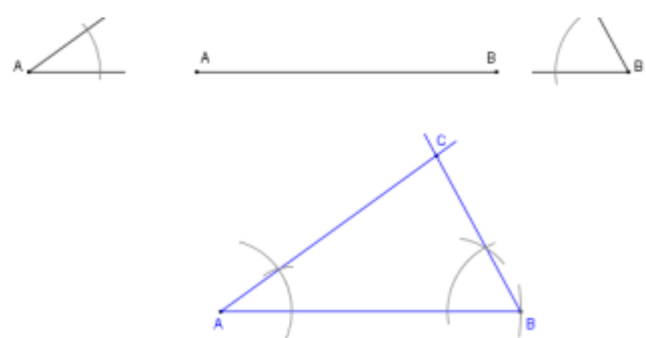
10. Set the compass width to the distance *AC*.



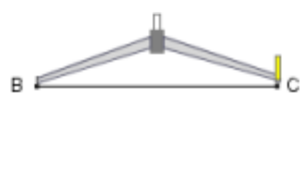
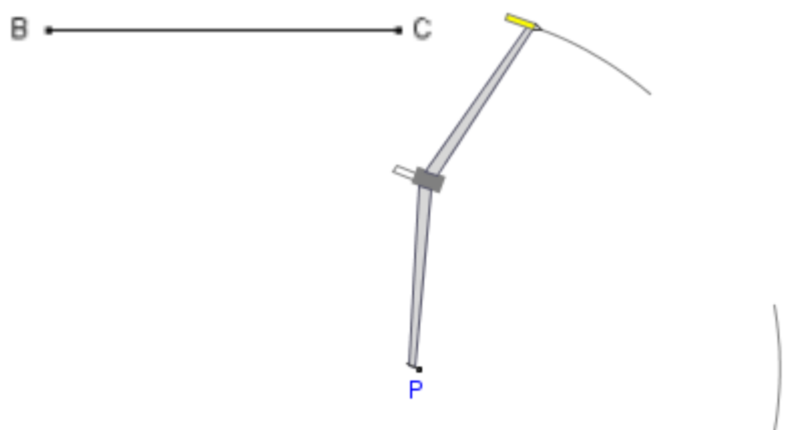
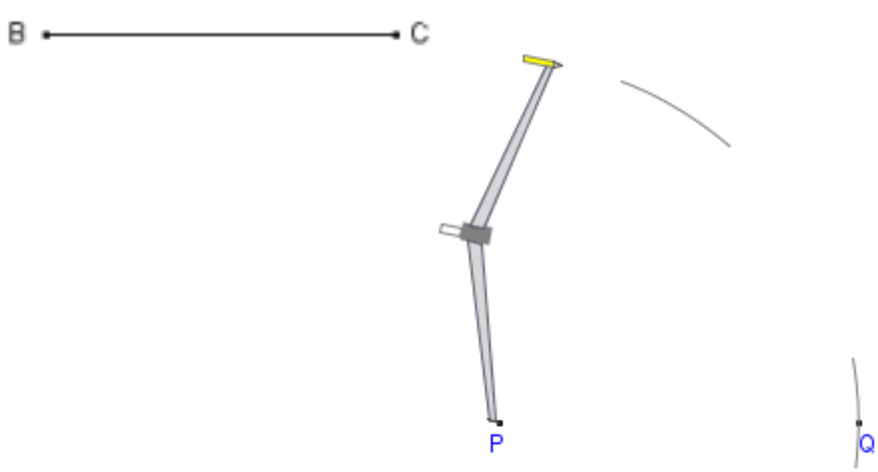
11. With the compass point on A, make an arc across the second ray, creating point C.	
12. Draw the segment $\overline{BC}$, the third side of the triangle	
Done! ΔABC has the desired two side lengths and included angle.	

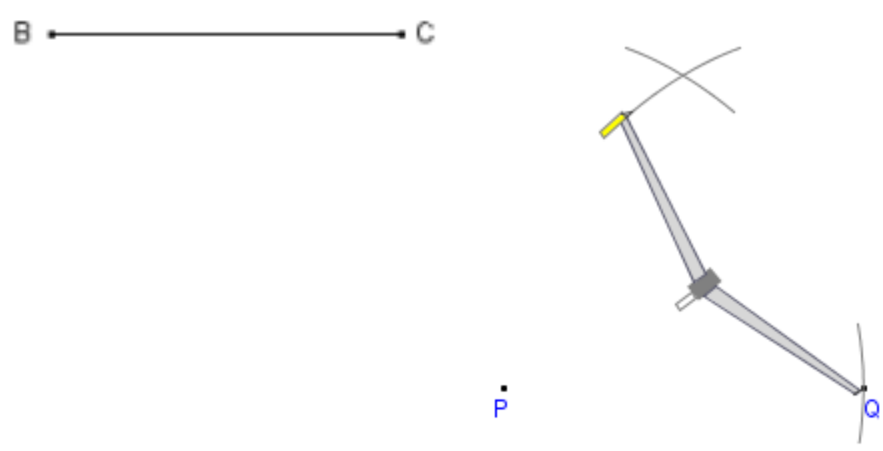
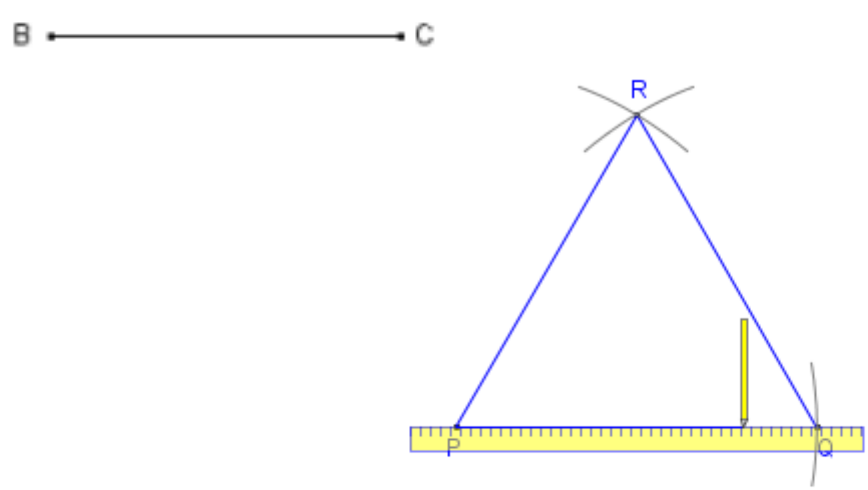
Example 3: **CONSTRUCTING A TRIANGLE GIVEN 2 ANGLES and the INCLUDED SIDE (ASA)**

After doing this	Your work should look like this
Start with the given line segment and two angles.	
1. Mark a point *A* that will be one vertex of the new triangle.	
2. Set the compass width to the length of $\overline{AB}$.	
3. With the compass point on *A*, make an arc near the future vertex *B* of the triangle.	
4. Mark a point *B* on this arc. Then draw segment $\overline{AB}$. This will be one side of the new triangle.	
5. With the compass at any width, draw an arc across both sides of the given angle.	

6. Without changing the compass width, draw an arc at point *A* on the new triangle. The arc must cross $\overline{AB}$ and also cross the future side of the triangle.	
7. Set the compass to the arc width at the given angle *A*. This is the distance between the points where the arc intersects the sides of the angle.	
8. Near point *A*, draw an arc in a similar position so it crosses the arc drawn earlier.	
9. Draw a line from *A* through the point where the arcs intersect. This will become the second side of the triangle. Draw it long.	
10. Repeat this process at *B*. Copy the angle measure from the given angle *B* to the new triangle at *B*. The point where the lines intersect is *C*, the third vertex of the triangle.	

Example 4: **CONSTRUCTING AN EQUILATERAL TRIANGLE**

After doing this	Your work should look like this
Start with the line segment $\overline{BC}$ which is the length of the sides of the desired equilateral triangle.	
1. Pick a point *P* that will be one vertex of the finished triangle.	
2. Place the point of the compass on point *B* and set its drawing end to point *C*. The compass is now set to the length of the sides of the finished triangle. Do not change it from now on.	
3. With the compass point on *P*, make two arcs, each roughly where the other two vertices of the triangle will be.	
4. On one of the arcs, mark a point *Q* that will be a second vertex of the triangle. It does not matter which arc you pick, or where on the arc you draw the point.	

5. Place the compass point on *Q* and draw an arc that crosses the other arc, creating point *R*.	
6. Using the straightedge, draw three segments linking the points *P*, *Q*, and *R*.	
7. Done! ΔPQR is an equilateral triangle. Its side length is equal to the distance BC.