Learning Targets:

Unit 5: Polynomials Functions

Lesson	Assignment	
5.1 Polynomial Functions		
Learning Targets:	Worksheet 5.1	
▪ I can evaluate polynomial functions.		
▪ I can identify general shapes of graphs of polynomials functions.		
5.2 Graphing Polynomial Functions		
Learning Targets:	Worksheet 5.2	
▪ I can graph polynomial functions and locate their real zeros.		
▪ I can find the maxima and minima of polynomial functions.		
5.3 Solving Equations Using Quadratic Techniques		
Learning Targets:	Worksheet 5.3	
▪ I can write expressions in quadratic form.		
radical form, and vice versa.		
▪ I can use quadratic techniques to solve equations.		
5.4 The Remainder and Factor Theorems		
Learning Targets:	Worksheet 5.4	
▪ I can evaluate functions using synthetic substitution.		
▪ I can determine whether a binomial is a factor of a polynomial by using synthetic substitution.		
5.5 Roots and Zeros		
Learning Targets:	Worksheet 5.5	
▪ I can determine the number and type of roots for a polynomial equati		
▪ I can find the zeros of a polynomial function.		
5.6 Operations on Functions		
Learning Targets:	Worksheet 5.6	
▪ I can find the sum, difference, product, and quotient of functions.		
▪ I can find the composition of functions.		
Unit 5: Review		

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ALGEBRA 2A **Name: ______________________________**
Lesson: 5.1 **Date: __________**

Polynomial functions

O b j e c t	❑ **I can evaluate polynomial functions** ❑ **I can identify general shapes of graphs of polynomial functions**

V o c a b u l a r y

Polynomial Function: only one variable (*x*)

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

a_n: ________________________________, ***not** zero*

a_0: ________________________________

n: ________________________________

All the exponents are _______________ numbers.

A polynomial function is in **standard form** if its terms are written in descending order of exponents from left to right.

Degree	Type	Standard Form
		$f(x) = a_0$
		$f(x) = a_1x + a_0$
		$f(x) = a_2x^2 + a_1x + a_0$
		$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$
	quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

Vocabulary

Value of a function: $f(k) =$

- One way to evaluate polynomial functions is to use **direct substitution**.
- Another way to evaluate a polynomial is to use **synthetic substitution**.

Real Zeros of a polynomial function: Maximum number of real zeros is equal to the degree.
Real zeros: where the graph crosses the x-axis.

How many zeros do the following functions have?

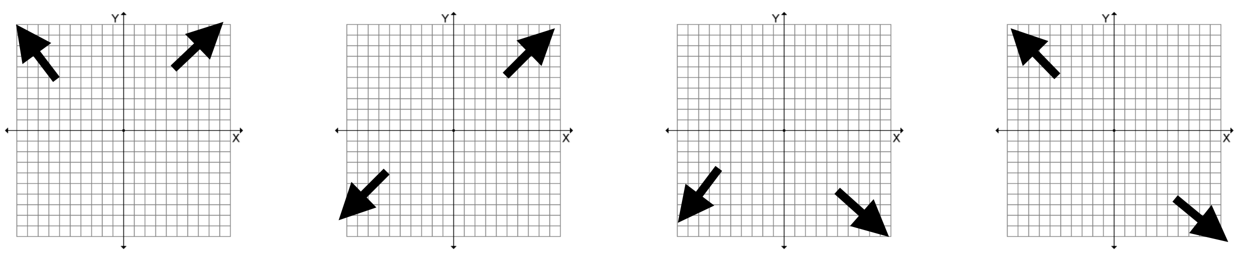
$f(x) = x + 2$ $g(x) = x^2 - 4$ $f(x) = x^3 - 2x^2 - 10x + 20$ $p(x) = x^4 + 8x^2 - 10$

______   ______   ______   ______

Zeros may be real or complex... **The Fundamental Theorem of Algebra:**

n*th *degree polynomial equation has* *__exactly__* *n* *solutions.

End behavior of a polynomial function:



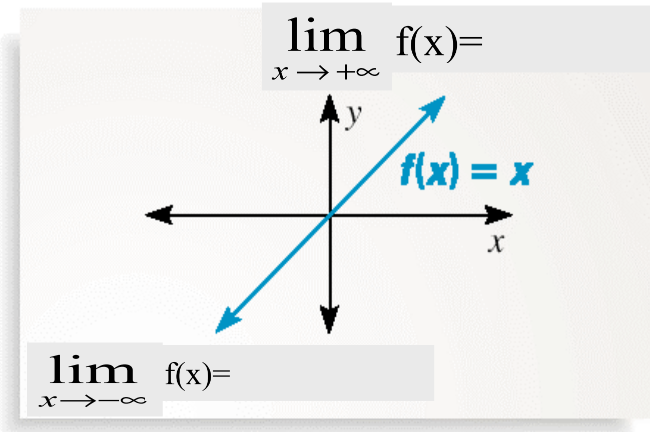
A   B   C   D

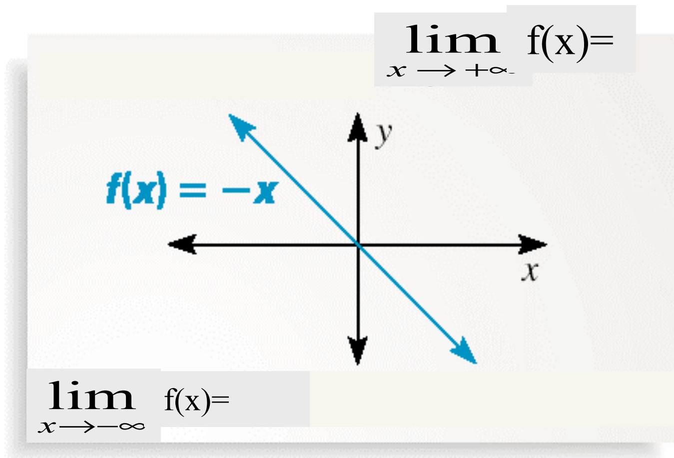
a_n	n	$x \to -\infty$	$x \to +\infty$
A) positive	even	$f(x) \to$	$f(x) \to$
B) positive	odd		
C) negative	even		
D) negative	odd		

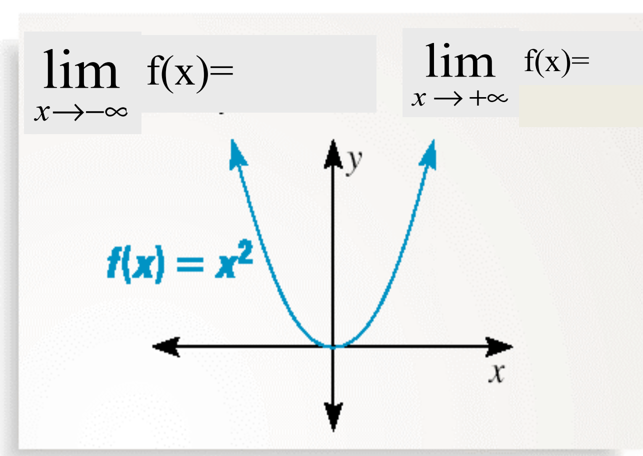
Instru	Example 1: Decide whether the function is a **polynomial function**. If it is, write the function in standard form and **state its degree**, type and **leading coefficient**. 1. $f(x) = \frac{1}{2}x^2 - 3x^4 - 7$ 2. $f(x) = 6x^2 + 2x^{-1} + x$ 3. $f(x) = x^3 + 3^x$ 4. $f(x) = -0.5x + \pi x^2 - \sqrt{2}$	
Your	Your Turn 1: *What are the degree and leading coefficient of* a) $3x^2 - 2x^4 - 7 + x^3$ b) $100 - 5x^3 + 10x^7$ c) $4x^2 - 3xy + 16y^2$ d) $4x^6 + 6x^4 + 8x^8 - 10x^2 + 20$	
Instruc	**Example 2:** Value of a function using **Direct Substitution** $f(x) = 2x^4 - 8x^2 + 5x - 7$ when $x = 3$. Solution:	**Example 3:** Value of a function using **Synthetic Substitution** $f(x) = 2x^4 - 8x^2 + 5x - 7$ when $x = 3$. Solution:
Your T	Your Turn 2: Use **direct substitution** *If* $f(x) = 2x^2 - 3x + 1$ *a)* ***f(-4)***	Your Turn 4: *Using **Synthetic Substitution.*** *Find* ***f(2)*** *a)* $3x^2 - 2x^4 - 7 + x^3$
	Your Turn 3: Use **direct substitution** *If* $f(x) = x^2 - 4x - 5$ *b)* $f(a^2-1)$	Your Turn 5: *Using **Synthetic Substitution.*** *Find* ***f(-5)*** ***b)*** $100 - 5x^3 + 10x^4$

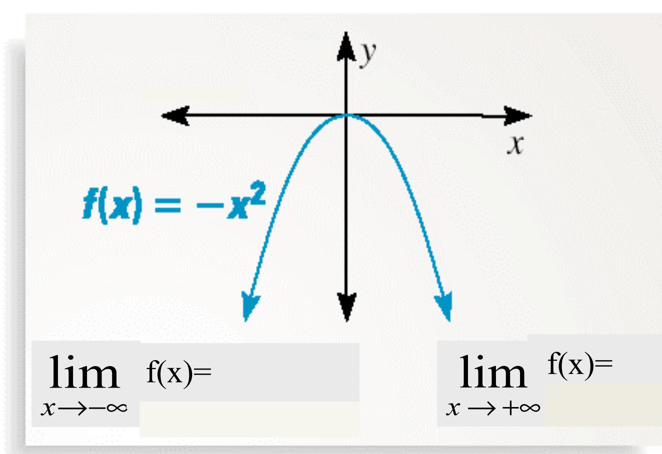
For each graph below,

- **describe the end behavior,**
- determine whether it represents an **odd-degree** or an **even-degree** function.
- **state the number of real zeros.**









Your Turn

Your Turn 6:

Degree: ________ # real zeros: _________

End behavior:

$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to +\infty} f(x) =$

Your Turn 7:

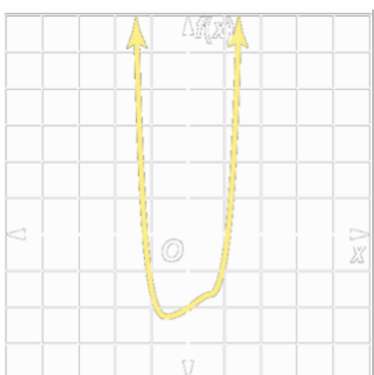
Degree: ________ # real zeros: _________

End behavior:

$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to +\infty} f(x) =$

Your Turn 8:



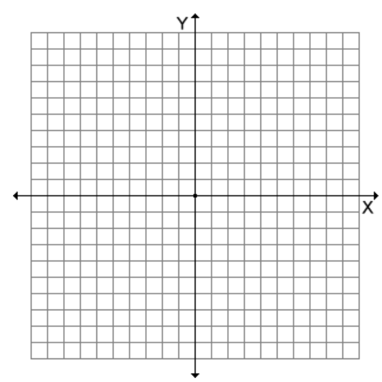
Degree: ________ # real zeros: _________

End behavior:

$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to +\infty} f(x) =$

Your Turn 9:
Your own:



Degree: ________ # real zeros: _________

End behavior:

$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to +\infty} f(x) =$

Closure 5.1

1. Give the degree and leading coefficient of each polynomial in one variable.

	degree	leading coefficient
a. $10x^3 + 3x^2 - x + 7$	________	________
b. $7y^2 - 2y^5 + y - 4y^3$	________	________
c. 100	________	________

Warm-up 5.1

1. State the degree and leading coefficient of $-4x^5 + 2x^3 - 6$.

Find $p(3)$ and $p(-5)$ for each function.

2. $p(x) = 12 - x^2$

p(3) = **p(-5) =**

3. $p(x) = x^3 - 10x + 40$

p(3) = **p(-5) =**

4. If $p(x) = x^2 - 3x + 4$, find $p(x + 2)$.

5. Determine whether the statement is *always, sometimes,* or *never* true.
A polynomial of degree three will intersect the x-axis three times.

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Lesson: 5.2 **Date:** __________

Graphing Polynomial Functions

O b j e c	❑ **I can graph polynomial functions and locate their real zeros** ❑ **I can find the maxima and minima of polynomial functions**
V o c a b u l a r y	*We have learned how to graph functions with the following degrees:* 0 *Example:* $f(x) = 2$ *horizontal line* 1 *Example:* $f(x) = 2x - 3$ *line* 2 *Example:* $f(x) = x^2 + 2x - 3$ *parabola* *How do you graph polynomial functions with degrees higher than 2?* ***We'll make a table of values, then graph...*** ***Graphs of Polynomial Functions:*** • *are continuous (there are no breaks)* • *have smooth turns* • *with degree n, have at most* ***n – 1*** *turns* • *Follows end behavior according to* ***n*** *(even or odd) and to* a_n *(positive or negative).*

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Example 1: **Graph by making a table of values and find the real zeros, x-coordinate of relative maxima & minima. Describe the end behavior of the graph.**

Example 2: **Graph by making a table of values and find the zeros, x-coordinate of relative maxima & minima. Describe the end behavior of the graph.**

$$f(x) = x^3 + x^2 - 4x - 1$$

$$f(x) = -x^4 - 2x^3 + 2x^2 + 4x.$$

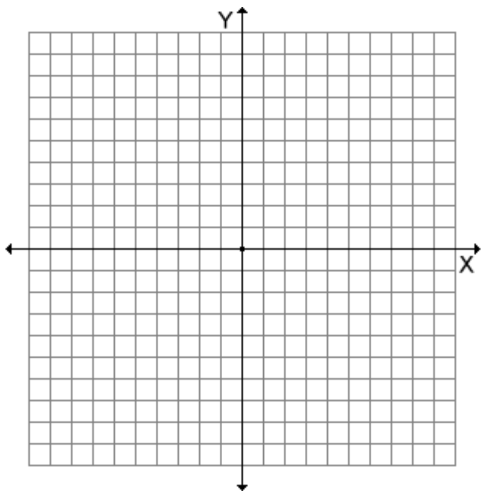
n: __ **a_n**: ____ **# turns**: at most _____ **# total zeros**: _____

n: __ **a_n**: ____ **# turns**: at most _____ **# total zeros**: _____

 # real zeros: at most ______

 # real zeros: at most

x	–3	–2	–1	0	1	2	3
f(x)							



End Behavior:

End Behavior: $\lim_{x \to -\infty} f(x) =$

$\lim_{x \to +\infty} f(x) =$

Real zeros: ______ (x-coordinate)

Zeros: between ____ and ____

between ____ and ____

between ____ and ____

Relative **maxima** & **minima**

Maxima at ___________________

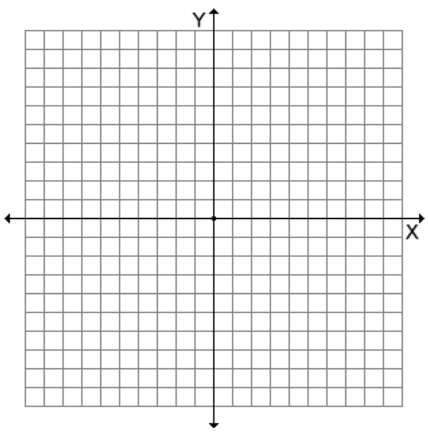
Minima at ____________________

Find the approximate zeros (to the nearest hundredths) ______________________________

Your Turn 1:

f(x) = 3x³ – 9x + 1

x	-3	-2	-1	0	1	2	3
f(x)							



$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to +\infty} f(x) =$

Relative **maxima** & **minima:**

Maxima at ____________________

Minima at ______________________

Real zeros: _______

Zeros: between ____ and ____

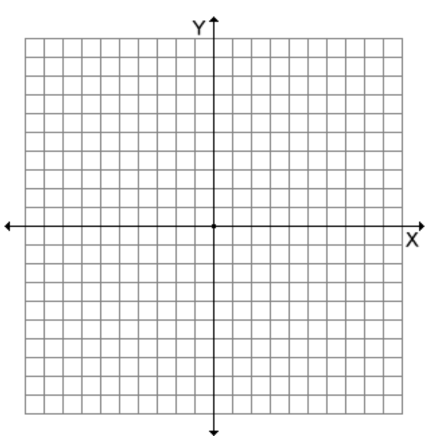
between ____ and ____

between ____ and ____

Your Turn 2:

f(x) = –x³ + 4x

x	-4	-3	-2	-1	0	1	2	3
f(x)								



$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to +\infty} f(x) =$

Relative **maxima** & **minima:**

Maxima at ____________________

Minima at ______________________

Real zeros: _______

Zeros: between ____ and ____

between ____ and ____

between ____ and ____

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Lesson: 5.3 Date: __________

Solving Equations by using Quadratic Techniques

<table>
<tr><td>Object</td><td>

- ❑ **I can write expressions in quadratic form**
- ❑ **I can use quadratic techniques to solve equations**

</td></tr>
<tr><td>Vocabulary</td><td>

Quadratic Form: | **Quadratic Formula:**

</td></tr>
<tr><td>Instruction</td><td>

Example 1: Write the given expression in quadratic form, if possible.

$$2x^6 + x^3 + 9$$

</td><td>

Example 2: Write the given expression in quadratic form, if possible.

$$7x^{10} + 6$$

</td></tr>
</table>

Instruction	***Example 3:*** Write the given expression in quadratic form, if possible. $$x^4 + 2x^3 - 1$$	***Example 4:*** Write the given expression in quadratic form, if possible. $$x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 4$$

Your Turn

In your own words: What is necessary for an expression to be written in quadratic form?

____

____

Your Turn 1: Write each expression in quadratic form, if possible.

a) $2x^4 + x^2 + 3$

b) $x^{12} + 5$

c) $x^6 + x^4 + 1$

d) $x - 2x^{1/2} + 3$

Example 5: Solve: $x^4 - 29x^2 + 100 = 0$

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Your Turn 2: Solve

$x^4 - 10x^2 + 9 = 0$

Example 7: Solve.

$$x^{\frac{1}{2}} - x^{\frac{1}{4}} - 6 = 0$$

Example 8: Solve.

$$x + \sqrt{x} = 12$$

Your Turn 3: Solve $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} + 6 = 0$

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Lesson: 5.4 **Date:** __________

The Remainder and Factor Theorems

Object	❑ **I can determine whether a binomial is a factor of a polynomial by using synthetic substitution**
Review	Use synthetic division: Example 1: $(2x^2 + 3x - 4) \div (x - 2)$ Example 2: $(p^3 - 6) \div (p - 1)$

Your Turn 1:

$(2x^3 - 7x^2 - 8x + 16) \div (x - 4)$

Factor Theorem

The binomial (x – a) is a factor of the polynomial *f(x)* if and only if *f(a) = 0*.

This means that the remainder of the synthetic division or long division is ________.

Example 3:

Show that x+5 is a factor of $x^3 + 2x^2 - 13x + 10$. Then find the remaining factors of the polynomial.

Example 4:

Given that (x+2) is a factor of *f(x)*, find the remaining factors of the polynomial

$$f(x) = x^3 - 13x^2 + 24x + 108$$

Your Turn 2:

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

1. $x^3 + x^2 - 10x + 8; x - 2$

2. $x^3 - 4x^2 - 11x + 30; x + 3$

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Lesson: 5.5 Date: __________

Roots and Zeros

O b j e c t	❑ **I can find all the zeros (real & imaginary) of a polynomial function** ❑ **I can find exact zeros by using the graphing calculator, synthetic substitution, and the Quadratic Formula**
I n s t r u c t i o n	see below

Types of Roots The following statements are equivalent for any polynomial function $f(x)$

- c is a zero of the polynomial function $f(x)$.
- $(x - c)$ is a factor of the polynomial $f(x)$.
- c is a root or solution of the polynomial equation $f(x) = 0$.

If c is real, then $(c, 0)$ is an intercept of the graph of $f(x)$.

A polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.

Complex Conjugate Theorem	Suppose a and b are real numbers with $b \neq 0$. If $a + bi$ is a zero of a polynomial function with real coefficients, then $a - bi$ is also a zero of the function.

Complex Zeros are always in pairs! A polynomial function may have ___ or ___ …or any ______ # of complex zeros.

Examples: ___________ & ____________ (its conjugate)
___________ & ____________ (its conjugate)
___________ & ____________ (its conjugate)

Example 1: Find all the zeros of. $f(x) = x^3 + x^2 + 9x + 9$

Step 1: Try some possible zeros by using **<u>synthetic substitution</u>**: *you may* ***<u>cheat</u>*** *with* ***Graph.Calc.***!

Step 2: Once you get a polynomial with degree 2 you can **solve the quadratic equation**!

Step 3: Give the *Answer:* Zeros are ________________________________

Example 2: Find all the zeros of *f(x)=* $x^4 - 21x^2 + 80$

Step 1: Try some possible zeros by using **<u>synthetic substitution</u>**: *you may* ***<u>cheat</u>*** *with* ***Graph.Calc.***!

Try another zero until you get a depressed polynomial with degree 2.

Step 2: Once you get a polynomial with degree 2 you can **solve the quadratic equation**!

Step 3: Give the *Answer:* Zeros are ________________________________

Your Turn 1: Find all the zeros of $f(x) = x^4 - 3x^3 + 21x^2 - 75x - 100$

Step 1:

Step 2:

Step 3: Answer ______________________________

Example 3: Write a polynomial function of least degree with integer coefficients whose zeros include 4 & $7i$ ⟶ ___________ (its conjugate)

Remember:

- ***Imaginary roots always come in pairs!!!***
- ***If p & q are roots of an equation, then (x-p) and (x-q) are factors!!!***

So, because there are ___ zeros, the least degree will be: _____. And we get the polynomial function with the least degree by multiplying:

Use FOIL or distributive property.
Hint: Drawing the arrows may help you to avoid mistakes!

Simplify by combining like terms.
Remember: $\boldsymbol{i^2 = -1}$

Answer:

Your Turn 2: Write a polynomial functions of least degree with integer coefficients whose zeros include 2 & $4i$. *Which one is missing?* _________

So, because there are ___ zeros, the least degree will be: _____. And we get the polynomial function with the least degree by multiplying:

Use FOIL or distributive property.

Simplify by combining like terms.

Answer:

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Lesson: 5.6

Name: ______________________________
Date: __________

Operations on Functions

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- ❑ **I can find the sum, difference, product, and quotient of functions.**
- ❑ **I can find the composition of functions.**

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Arithmetic Operations

Operations with Functions	Sum	$(f+g)(x) = f(x) + g(x)$
	Difference	$(f-g)(x) = f(x) - g(x)$
	Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
	Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$

Composition of Functions

There is a 40% off sale at Old Navy and as an employee you receive a 10% discount, how much will you pay on a $299 jacket?

You do not get 50% off…

...this is an example of a composite function.

You will pay 90% of the cost (10% discount) after you pay 60% (40% discount). The two functions look like this…

f(x) = 0.9x ***g(x) = 0.6x***

We can put these together in a composite function that looks like this…

f(g(x))
"f of g of x"

Instruction	Example 1: *restriction: g(x) ≠ 0 because:*____________________
Your Turn	Your Turn 1: *Don't forget the restriction since the denominator cannot ever be equal to ___!*

Instruction	Example 2: **Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.** $[g \circ h](x) = g[h(x)]$ $\quad\quad$ $[h \circ g](x) = h[g(x)]$
	Example 3: *If $f(x) = x^2 - 5$ and $g(x) = 3x^2 + 1$* *find* ***f[g(2)]*** $\quad\quad$ *find* ***g[f(2)]***
Your Turn	Your Turn 2: **Find $[f \circ g](x)$ and $[g \circ f](x)$.** $f(x) = 2x + 7; g(x) = -5x - 1$
	Your Turn 3: **Find $[f \circ g](x)$ and $[g \circ f](x)$.** $f(x) = x^2 - 1; g(x) = -4x^2$