Unit 5: Transformations
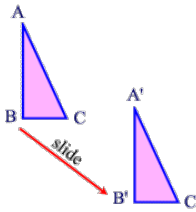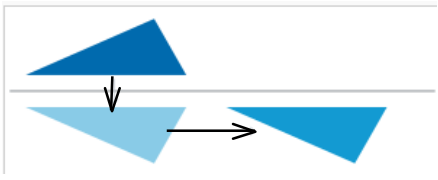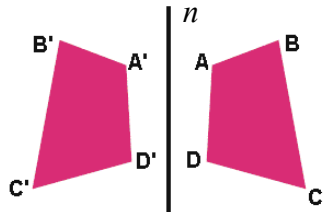
5.1 Reflections

Targets

- ○ I can draw reflected images.

- ○ I can recognize and draw lines of symmetry and points of symmetry.

Instruction (Vocabulary)

Term/Concept	Definition/Example	Picture
Transformation	○ A **transformation** maps an initial figure called a _____ onto a final figure called an _____.	

Types of Transformations

1. _____
*A figure can be **flipped** over a line.

2. _____
*A figure can be **slid** in any direction.

3. _____
*A figure can be **turned** around a point.

4. _____
*A figure can be **flipped** and **slid**.

Instruction (Vocabulary)

Term/Concept	Definition/Example	Picture
Reflection	A **reflection** is a transformation representing a _____ of a figure.	
Notations for reflections	_____ means, "Reflect ABCD over line n"	

Example 1:

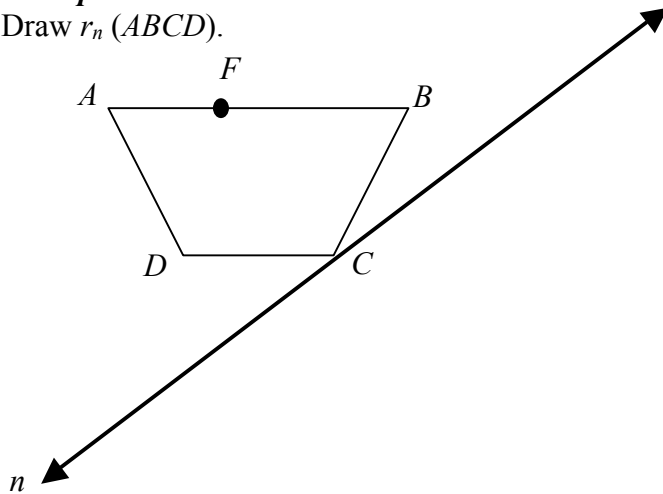Draw $r_m(\Delta SUN)$.   (Reflect ΔSUN over line m.)
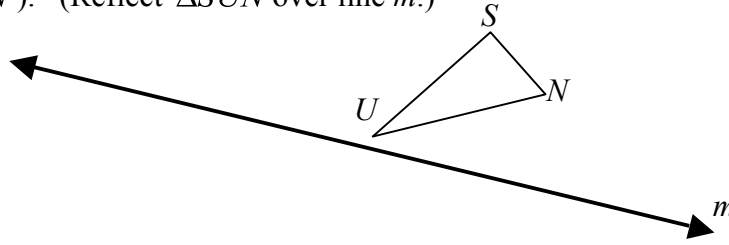
Based on your drawing, answer the following questions:

1. Draw a line from N to N'. What is the relationship between $\overline{NN'}$ and m?

2. Measure the distance from N to line m and N' to line m. What do you notice?

3. The orientation of ΔSUN is counter-clockwise. What is the orientation of $\Delta S'U'N'$?

Example 2:

Draw $r_n(ABCD)$.

Based on the drawing above, answer the following questions:

1. Measure $\angle A$ and $\angle A'$. What do you notice?

2. In the pre-image, F is between A and B. Where is F' in your image?

3. A, F, and B are collinear in the pre-image. Which points are collinear in your image?

4. Find the distance between A and D. How does it compare to $A'D'$?

Instruction

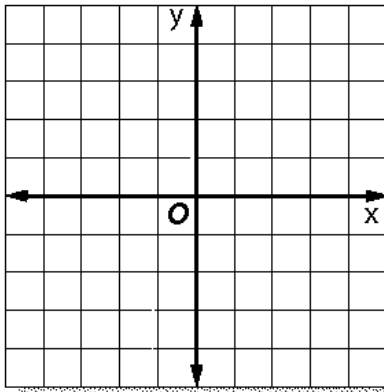| **Congruence Transformation (Isometry):** A transformation that is a _____ or a _____ of _____.	**ABCD Theorem**: All congruence transformations preserve the following properties: A – B – C – D –

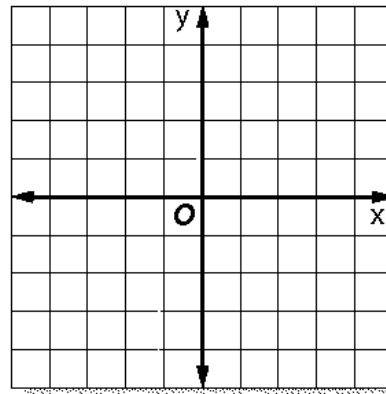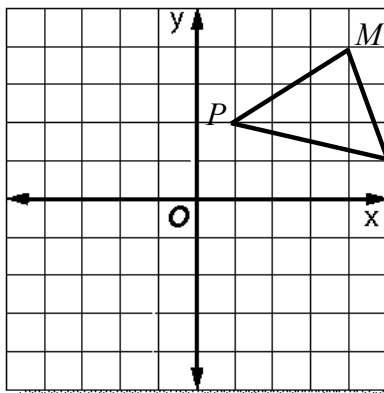| **Example 3:** Graph $\triangle CAT$ with vertices $C(-2, 5)$, $A(1, -2)$, $T(3, 4)$. Draw $r_{\,x\text{-axis}}(\triangle CAT)$ and record the coordinates of the image. $C'(\quad, \quad)$ $A'(\quad, \quad)$ $T'(\quad, \quad)$	**Your turn:** Graph $JADE$ with vertices $J(4, -1)$, $A(2, 4)$, $D(0, 2)$, $E(2, -5)$. Draw $r_{\,y\text{-axis}}(JADE)$ and record the coordinates of the image. $J'(\quad, \quad)$ $A'(\quad, \quad)$ $D'(\quad, \quad)$ $E'(\quad, \quad)$

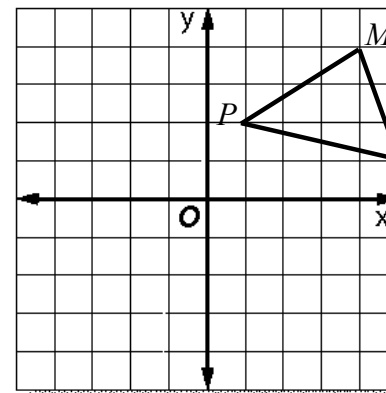| **Example 4:** $\triangle MNP$ has vertices $M(4, 4)$, $N(5, 1)$, $P(1, 2)$. Draw $r_{\,x\,=\,2}(\triangle MNP)$ and record the coordinates of the image. $M'(\quad, \quad)$ $N'(\quad, \quad)$ $P'(\quad, \quad)$	**Your turn:** $\triangle MNP$ has vertices $M(4, 4)$, $N(5, 1)$, $P(1, 2)$. Draw $r_{\,y\,=\,-1}(\triangle MNP)$ and record the coordinates of the image. $M'(\quad, \quad)$ $N'(\quad, \quad)$ $P'(\quad, \quad)$

5.2 Translations

Targets	o I can draw translated images using coordinates. o I can draw translated images by using repeated reflections.

<table>
<tr><td rowspan="4">Instruction (Vocabulary)</td><td>Term/Concept</td><td>Definition/Example</td><td>Picture</td></tr>
<tr><td>Translation</td><td>A <u>translation</u> is a transformation that moves all points of a figure the same distance in the same direction.</td><td>A

B   C   A'
slide
B'   C'</td></tr>
<tr><td>Notation for translations</td><td>_____ means slide
ΔABC _____ units to the right/left and _____ units up/down</td><td></td></tr>
<tr><td colspan="3">

Example 1:
Graph ΔABC with $A(-3, 5)$, $B(1, -2)$, and $C(-2, -3)$ and its image under the translation $T_{(3, -1)}$.

Your Turn:
Graph $TWXYZ$ with $T(4, -1)$, $W(3, -4)$, $X(-2, -5)$, $Y(-1, 1)$, and $Z(2, 2)$ and its image under the translation $T_{(-2, 0)}$.

</td></tr>
</table>

Instruction

Example 2:
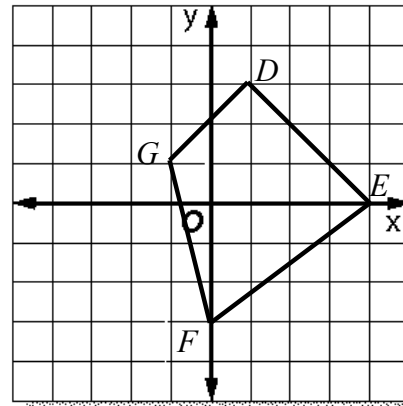*DEFG* has vertices *D*(1, 3), *E*(4, 0), *F*(0, -3), and *G*(-1, 1). Graph the image of *DEFG* under the translation
$(x, y) \rightarrow (x - 4, y + 1)$.

$D(1, 3) \rightarrow D'$

$E(4, 0) \rightarrow E'$

$F(0, -3) \rightarrow F'$

$G(-1, 1) \rightarrow G'$

Your Turn:
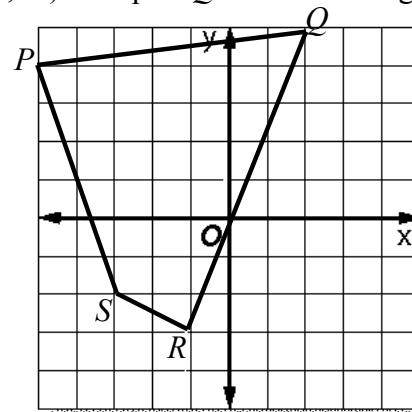*PQRS* has vertices *P*(-5, 4), *Q*(2, 5), *R*(-1, -3), and *S*(-3, -2). Graph *PQRS* and its image under the translation
$(x, y) \rightarrow (x + 3, y - 2)$.

$P(-5, 4) \rightarrow P'$

$Q(2, 5) \rightarrow Q'$

$R(-1, -3) \rightarrow R'$

$S(-3, -2) \rightarrow S'$

Example 3:
Identify the translation in both words and symbols that moved each figure.

a. figure 1 → figure 3

words: _____

symbols: _____

b. figure 3 → figure 2

words: _____
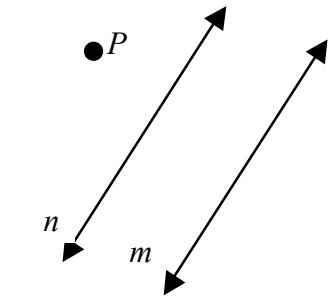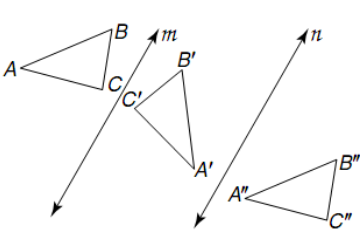
symbols: _____
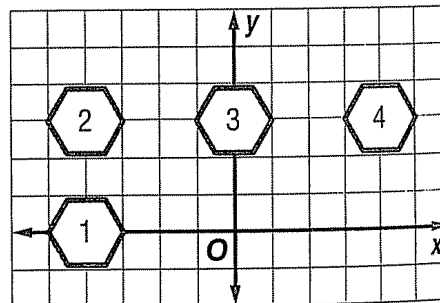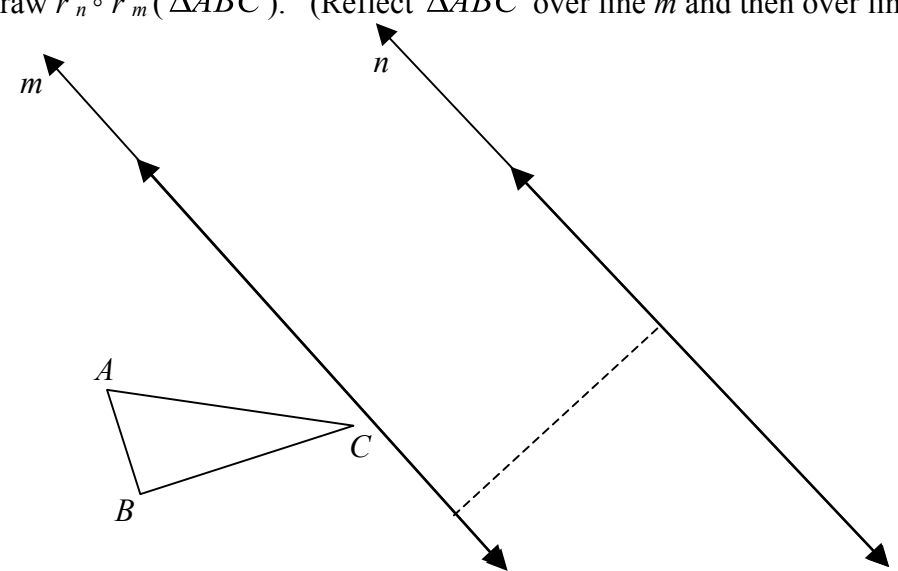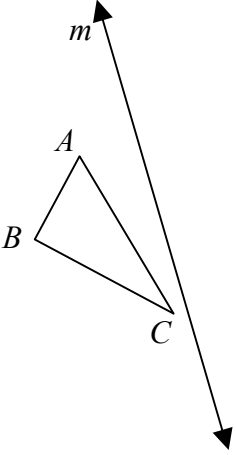
c. figure 4 → figure 1

words: _____
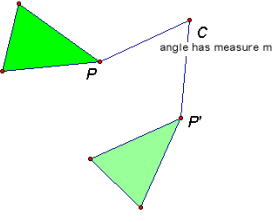
symbols: _____

Instruction (Vocabulary)

Term/Concept	Definition/Example	Picture
Composite of reflections	If a point P is reflected over line n, and then P' is reflected over line m, we can write this as a composite of the reflections: _____ or _____	
Translation by Repeated Reflections	A **translation by repeated reflections** is a transformation in which a figure is reflected twice over _____. $r_n \circ r_m (\Delta ABC)$ is a translation only if lines m and n are _____.	

Example 5:

Draw $r_n \circ r_m (\Delta ABC)$. (Reflect ΔABC over line m and then over line n)

Based on the figures above, answer the following questions:

1. Measure the distance from *B* to *B*". _____

2. Measure the "dashed" line from *m* to *n*. _____

3. What is the relationship between the two distances? _____

4. ΔABC has a counter-clockwise orientation. What is the orientation of $\Delta A"B"C"$?

Term/Concept	Definition/Example
Magnitude of a Translation	o The **magnitude of a translation** is the _____ between any point and its image.

Example 6:

Translate ΔABC 4 cm by adding a second reflecting line to the figure.

5.3 Rotations

Targets	o I can draw rotated images using the angle of rotation. o I can identify the order and magnitude of a rotational symmetry.

Term/Concept	Definition/Example	Picture
Rotation	o A **rotation** is a transformation that _____ every point of a pre-image through a specified _____ and _____ about a fixed _____.	
Notation for Rotations	$R_{90°}$ means rotate a figure 90° _____ around _____ $R_{-90°}$ means rotate a figure 90° _____ around _____	

Instruction

Example 1:

Draw R$_{-90°}$(ΔABC). ΔABC has vertices A(-2, -4), B(-1, 3), and C(2, 1). Then state the coordinates of the image of ΔABC.
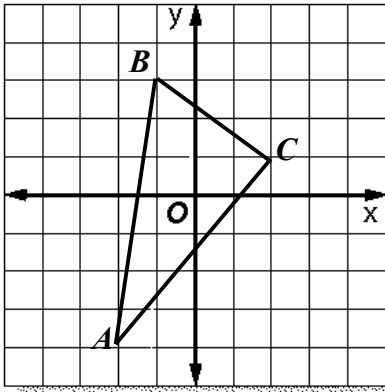
A'(,   )

B'(,   )

C'(,   )

Example 2:

Draw R$_{90°}$(ΔABC). ΔABC has vertices A(-2, -4), B(-1, 3), and C(2, 1). Then state the coordinates of the image of ΔABC.
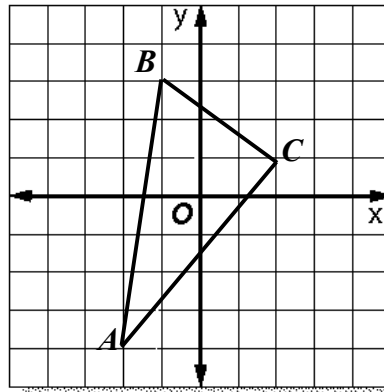
A'(,   )

B'(,   )

C'(,   )
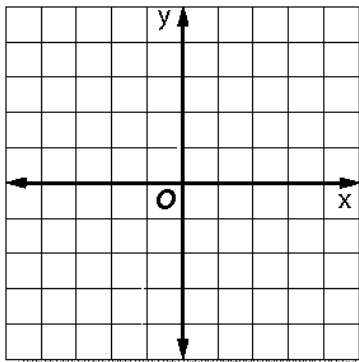
Rotation Rules:

R$_{-90°}$ maps the point $(x, y) \rightarrow$ _____

R$_{90°}$ maps the point $(x, y) \rightarrow$ _____

Your Turn:

Draw ΔDEF with vertices D(-3, -1), E(2, -4), and F(1, 2). Draw R$_{-90°}$(ΔDEF). Then state the coordinates of the image of ΔDEF.

D'(,   )

E'(,   )

F'(,   )

# Using Repeated Reflections to Rotate Figures

Instruction

Example 4:

Given $\triangle JET$ and reflection line *m*, rotate $\triangle JET$ by 130 degrees by adding a 2nd reflection line.

Step 1: Using the point on the given reflection line as the vertex, measure an angle that is HALF the measure of the angle of rotation specified.

Step 2: Extend the angle so that it forms a line that intersects the given (1st) reflection line. The new line formed is the 2nd reflection line. Label it line *n*.

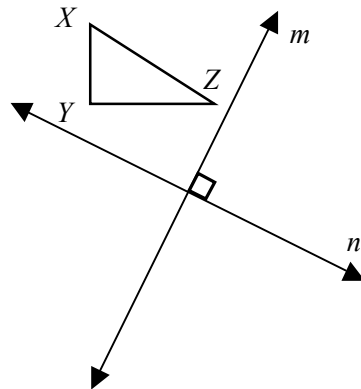**Step 3:** Reflect the figure over the original (1st) reflection line.

Step 4: Reflect the image from step 3 over the 2nd reflection line.
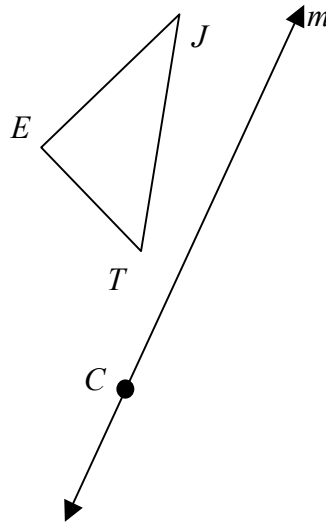
Based on the figure above, answer the following questions:

1. Draw in and measure $\angle TCT"$. _____.

2. How does the measure of $\angle TCT"$ compare to your angle of rotation?

3. $\triangle JET$ has a counter-clockwise orientation. What is the orientation of $\triangle J"E"T"$?

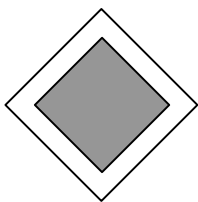*Example 5:* Draw $r_n \circ r_m (\triangle XYZ)$. What is the magnitude of the resulting rotation?

Term/Concept	Definition/Example	Picture
Rotational Symmetry	o A figure has **rotational symmetry** if it can be rotated less than or equal to 360° about a point so that the _____ looks exactly like the _____.	
Order	o The **order** of a rotational symmetry is the number of _____ less than or equal to 360° that produce an image identical to the original. o "how many matches there are as you go around once" o the number of positions in which the object looks exactly the same	Order: _____ Magnitude: _____
Magnitude of a Rotational Symmetry	o The **magnitude** of a rotational symmetry figure is	

Instruction (Vocabulary)

Example 6:
Identify the order and magnitude of the rotational symmetry of each figure.
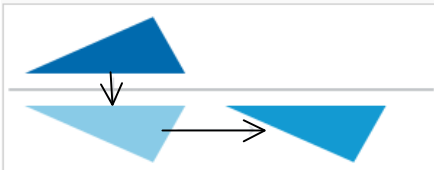
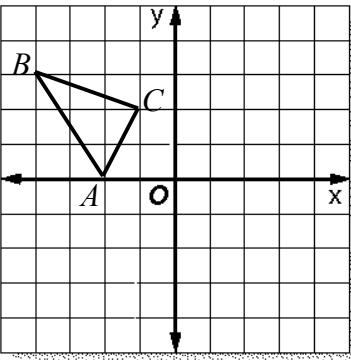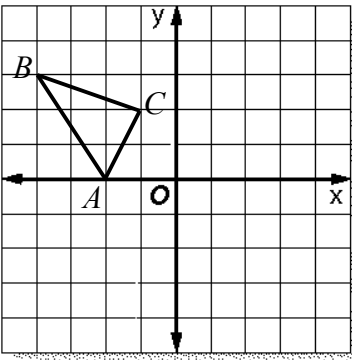|			
Order	Order	Order	Order
Magnitude	Magnitude	Magnitude	Magnitude

5.4 Glide Reflections

Targets

- o   I can draw images using glide reflections.
- o   I can identify composites of transformations

Instruction (Vocabulary)

Term/Concept	Definition/Example	Picture
Glide Reflection	A **glide reflection** is a transformation that is a _____ of two transformations: a _____ and a _____ .	

Instruction

Example 1:
ΔABC has vertices $A(-2, 0)$, $B(-4, 3)$, and $C(-1, 2)$. Draw $r_{\text{y-axis}} \circ T_{(0, -4)}$. In other words :
translation: $(x, y) \rightarrow (x, y - 4)$, then
reflection: over the y - axis

$A''(\quad , \quad)$

$B''(\quad , \quad)$

$C''(\quad , \quad)$

Example 2:
ΔABC has vertices $A(-2, 0)$, $B(-4, 3)$, and $C(-1, 2)$. Draw $r_{\text{x-axis}} \circ T_{(3, 0)}$.

$A''(\quad , \quad)$

$B''(\quad , \quad)$

$C''(\quad , \quad)$

Your turn:
Repeat example 2, but reverse the order of the transformations. Draw $T_{(3, 0)} \circ r_{\text{x-axis}}$.

$A''(\quad , \quad)$

$B''(\quad , \quad)$

$C''(\quad , \quad)$

Does reversing the order of the transformations change the image?

Example 3:

Draw $r_k \circ r_n \circ r_m (\Delta ABC)$.
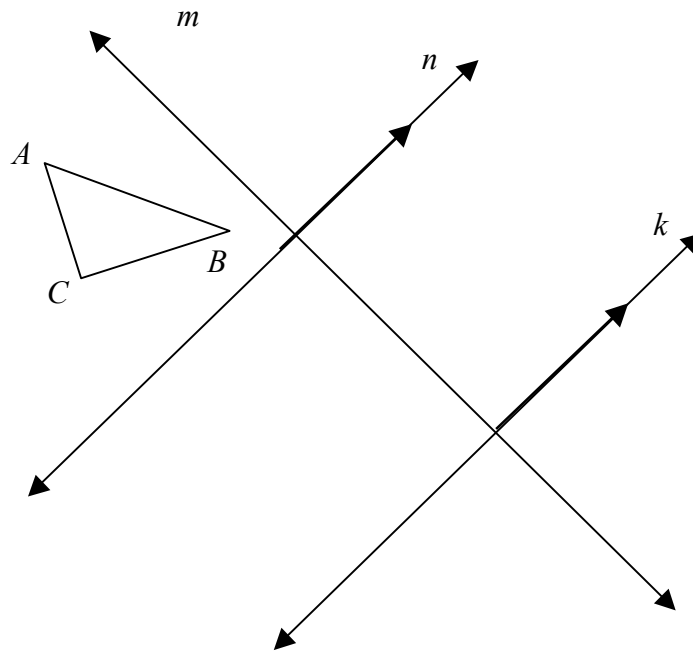
a. What type of transformation results from this composition of reflections?

b. Which of the following are preserved?

   Angle measure? _____   Betweenness? _____   Collinearity? _____   Distance? _____

c. What is the orientation of the preimage ? _____

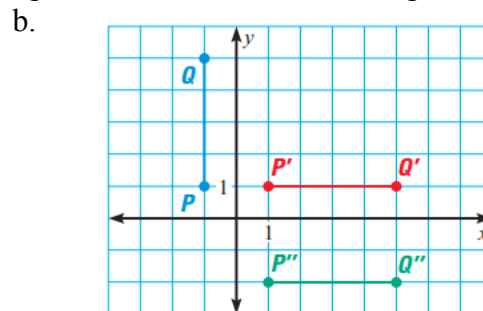d. What is the orientation of the **final** image ? _____

Example 4:

Describe the composition of transformation in each figure below in words and using composite notation.
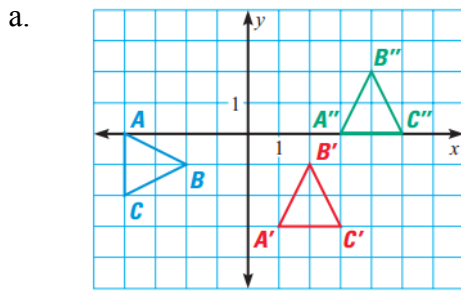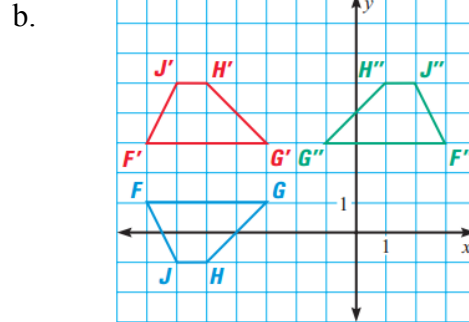
a.

b.
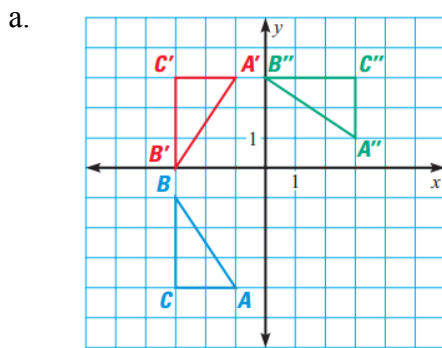
Words:

Notation:

Words:

Notation:

Your turn:

a.

b.

Words:

Words:

Notation:

Notation:

Example 5:

MC Escher is one of the most famous graphic artists. In his lifetime (1898-1972) he made over 2000 sketches, and two of his works are shown below.

Using your colored pencils,
   a.   Shade in **red** an example of a **rotation**.
   b.   Shade in **blue** an example of a **translation**.
   c.   Shade in **green** an example of a **glide reflection**.