Lesson 6.1: Inverse Functions and Relations

Learning Targets:

I can find the inverse of a function or relation.
I can determine whether two functions or relations are inverses.

Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

- **<u>Relation</u>** – a mapping of input values (x-values) onto output values (y-values).

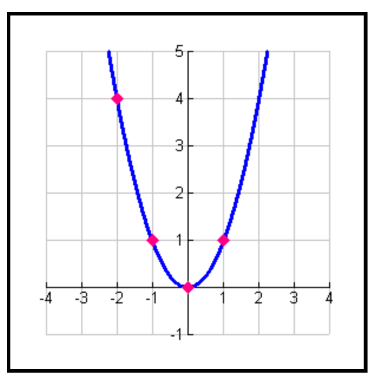
Here are 4 ways to show the same relation.

Equation:

$y = x^2$

Table of values:

Graph:



Mapping:

Is this relation a function?

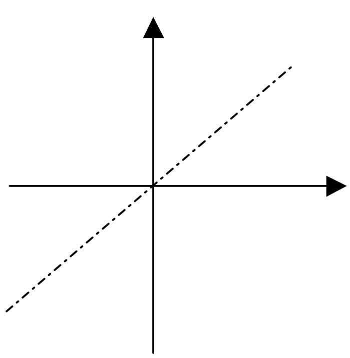
____

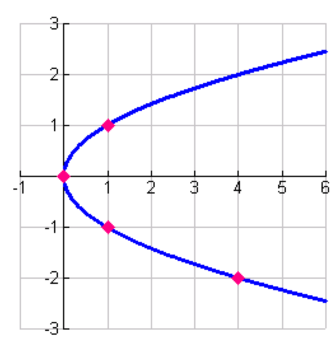
<u>Inverse relation</u> – just think: switch the x & y-values.

Equation: switch the x and y & **solve for y**

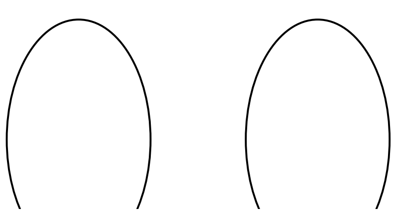
Table: switch the columns

Graph: The reflection of the original graph on the line ___________





Mapping: switch the domain & range



Is this relation a function?

____

<u>Example 1:</u> Find the inverse of the function $f(x)=\frac{2}{5}x-\frac{1}{5}$**.**

Step 1: Replace ***f(x)*** with **y** in the original equation.

Step 2: Interchange x and y.

Step 3: **Solve for y**.

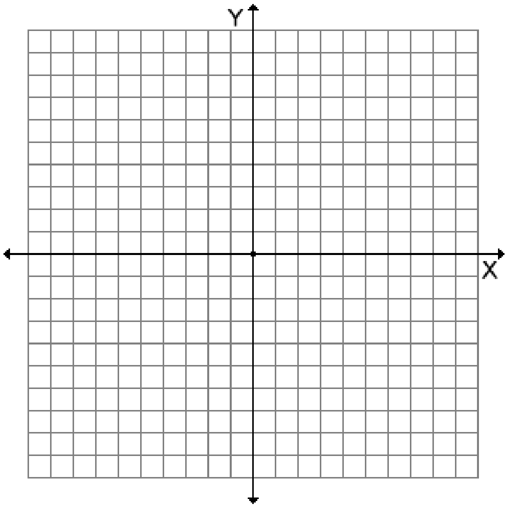
Step 4: Replace y with $f^{-1}(x)$

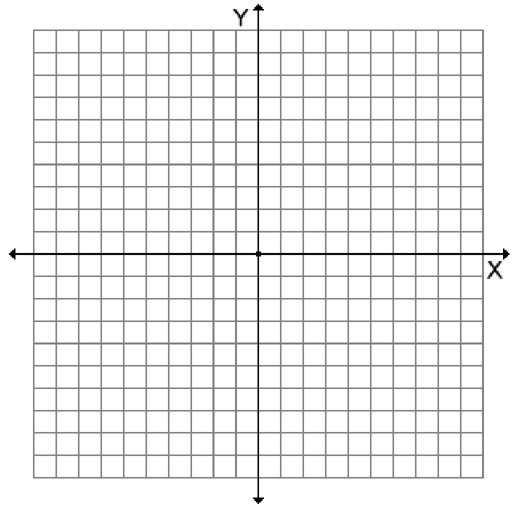
$f^{-1}(x)$ means "f inverse of x"

Find the inverse of each function. Then graph the function <u>and</u> its inverse.

1. $f(x)=\frac{2}{3}x-1$

2. $f(x)=2x-3$





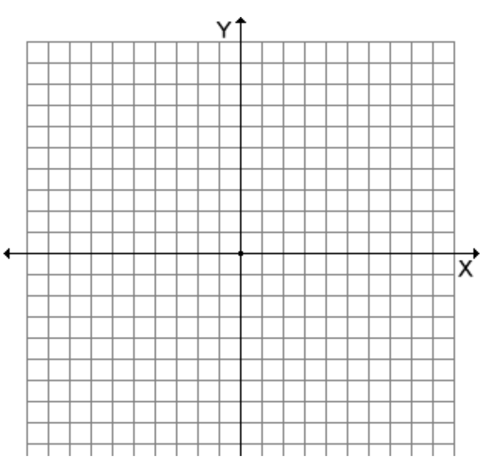
*How are the two lines (in each graph) related?*________________________________

Lesson 6.2: Exponential Functions

Learning Targets:

- I can graph an exponential function.
- I can determine if a function is growth or decay.
- I can write an exponential function given values.
- I can solve exponential functions.

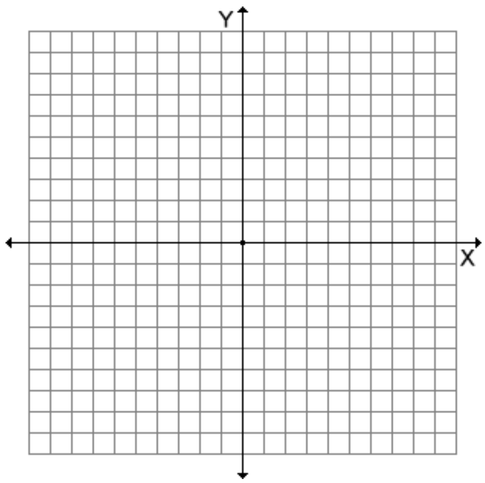
Example 1 Sketch the graph of $y = 4^x$ and identify its domain and range.



Domain:___________

Range:____________

Example 2 Sketch the graph of $y = 0.7^x$ and identify its domain and range.



Domain:___________

Range:____________

Example 3 Indicate whether each shows exponential growth or decay.

$y = 0.7^x$ $\quad$ $y = \frac{1}{3}(2)^x$ $\quad$ $y = 10\left(\frac{2}{5}\right)^x$

Example 4 Write an exponential function whose graph passes through the given points.

(0, -2) and (3, -54)

Example 5 Write an exponential function whose graph passes through the given points.

(0, 7) and (1, 1.4)

Example 6 Write an exponential function whose graph passes through the given points.

(0, 3) and (-1, 6)

Example 7 Write an exponential function whose graph passes through the given points.

(0, -18) and (-2, -2)

Example 8 Simplify the expressions below.

a) $5^{\sqrt{3}} \div 5^{\sqrt{2}}$

b) $\left(6^{\sqrt{5}}\right)^{\sqrt{6}}$

c) $2^{\sqrt{5}} \div 2^{\sqrt{3}}$

d) $\left(7^{\sqrt{3}}\right)^{\sqrt{7}}$

Example 9 Solve the equation.

$$4^{9n-2} = 256$$

Example 10 Solve the equation.

$$3^{5x} = 9^{2x-1}$$

Example 11 Solve the equation.

$$2^{3x+1} = 32$$

Lesson 6.3: Logarithmic Functions

Learning Targets:

- I can convert from logarithmic to exponential form and vice versa.
- I can evaluate logarithmic expressions.
- I can solve logarithmic equations.
- I can graph a logarithmic function.

Definition of Logarithm:

Let $b > 0$ and $b \neq 1$. Then n is the logarithm of m to the base b, written

$$\log_b m = n \quad \text{if and only if} \quad b^n = m$$

Check it out!:

Exponential Form		**Logarithmic Form**
$2^4 = 16$	means	______________________
$2^3 = 8$	means	______________________
$2^2 = 4$	means	______________________
$2^1 = 2$	means	______________________
$2^0 = 1$	means	______________________
$2^{-1} = \frac{1}{2}$	means	______________________
$2^{-2} = \frac{1}{4}$	means	______________________

Example 1: Convert to exponential form.

a) $\log_3 9 = 2$ b) $\log_{10} \frac{1}{100} = -2$ c) $\log_9 81 = 2$ d) $\log_3 \frac{1}{9} = -2$

Flower Power Root Rule: ***Algebra 1 Refresher***

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$$

Example:

$$16^{\frac{3}{4}} = ?$$

Example 2: Convert to logarithmic form.

a) $5^3 = 125$ b) $27^{\frac{1}{3}} = 3$ c) $3^4 = 81$ d) $81^{\frac{1}{2}} = 9$

Example 3: Evaluate logarithmic expressions.

a) $\log_3 243$ b) $\log_{10} 1000$ c) $\log_9 9^2$ d) $7^{\log_7(x^2-1)}$

Think-Pair-Share!

a) $\log_5 5^3$ b) $3^{\log_3(x+2)}$

Example 4: Solve logarithmic equations.

a) $\log_8 n = \frac{4}{3}$ b) $\log_{27} n = \frac{2}{3}$ c) $\log_4 x^2 = \log_4(4x-3)$ d) $\log_5 x^2 = \log_5(x+6)$

Example 5: Graph the logarithmic function. $x = \log_2(y)$

Step 1:
Convert the logarithmic form to exponential form.

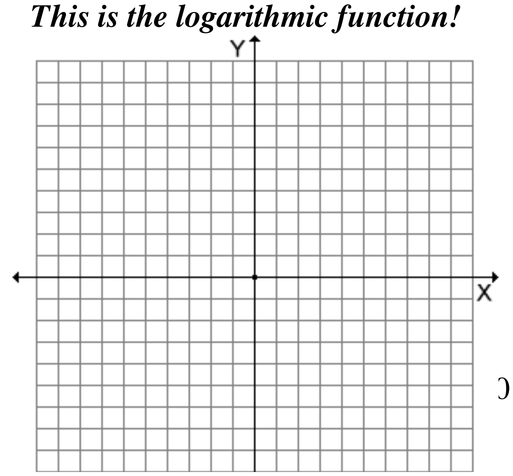
Step 2: Complete the table of values for the function in *exponential form*.

2	
x	**y**
3	
-2	
-1	
0	
1	

Step 3: Find the **inverse** of the coordinates.

x	**y**

Step 4: Graph the **inverse points**
This is the logarithmic function!



What is the x-intercept?: ____

<u>Example 6:</u> Graph the logarithmic function.

$$x = \log_{0.5}(y)$$

Step 1:
Convert the logarithmic form to exponential form.

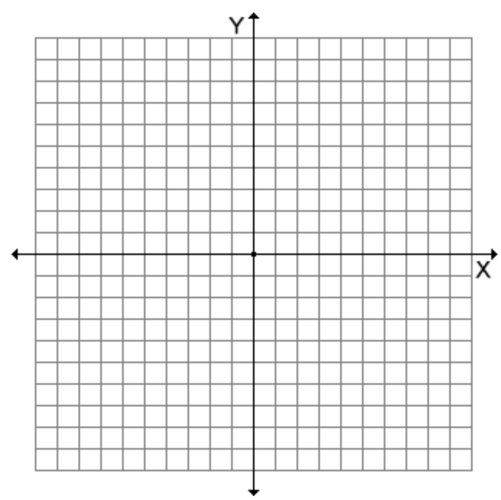
Step 2: Complete the table of values for the function in *exponential form*.

x	y

Step 3: Find the **inverse** of the coordinates.

x	y

Step 4: Graph the **<u>inverse points</u>**
This is the logarithmic function!



What is the x-intercept?: ____

<u>Your turn!</u>

Graph the logarithmic function. $x = \log_{1.7}(y)$

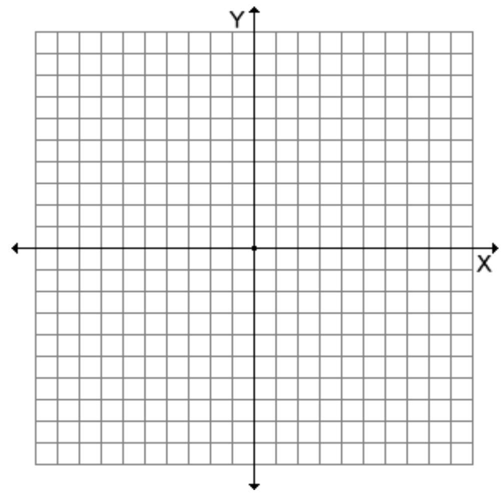
Step 1:
Convert the logarithmic form to exponential form.

Step 2: Complete the table of values

x	y
-2	
-1	
0	
1	
2	

Step 3: Find the **inverse** of the coordinates.

x	y



Step 4: Graph the **<u>inverse points</u>** for the function in *exponential form*.
This is the logarithmic function!

What is the x-intercept?: ____

Lesson 6.4: Properties of Logs

Learning Targets:

- I can use the product and quotient properties of logs.
- I can use the power property of logs.
- I can solve equations using properties of logs.

Product Property: $\log_b(x \cdot y) = \log_b x + \log_b y$

Example:

Quotient Property: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Example:

Power Property: $\log_b(x^n) = n \cdot \log_b x$

Example:

Example 1: Solve: $4 \log_2 x - \log_2 5 = \log_2 125.$

Example 2: Solve: $\log_8 x + \log_8 (x - 12) = 2.$

Your Turn: Solve each equation.

a) $2 \log_3 x - 2 \log_3 6 = \log_3 24$

b) $\log_2 x + \log_2 (x - 6) = 4$