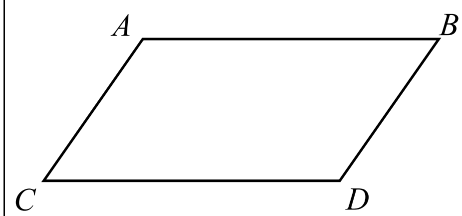
Unit 6: Quadrilaterals

6.1 Properties of Parallelograms

Targets

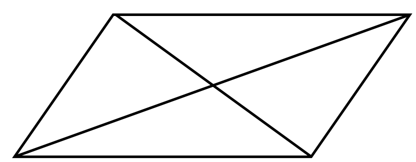
- I can recognize and apply properties of the sides and angles of parallelograms.
- I can recognize and apply properties of the diagonals of parallelograms.

Instruction

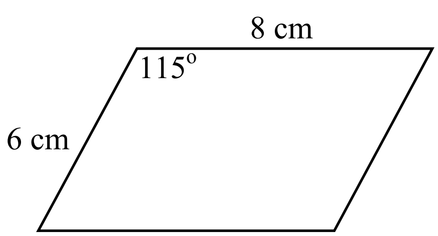
Term/Concept	Definition/Example	Picture
Parallelogram	A **parallelogram** is a __________________ where both pairs of opposite sides are _______________.	

Properties of parallelograms:

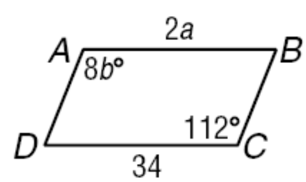
- Opposite sides are ____________________________
- Opposite sides are ____________________________
- Opposite angles are ____________________________
- Consecutive angles are __________________________
- Diagonals ____________________________________



Example 1:
Find all the missing side and angle measures in the parallelogram below.



Example 2:
If *ABCD* is a parallelogram, find the values of *a*, *b*, and $m\angle B$.



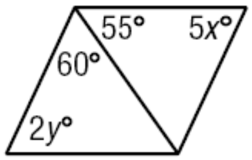
$a =$ _____

$b =$ _____

$m\angle B =$ _____

Instruction

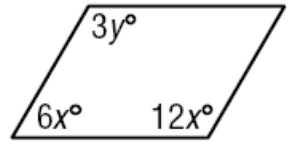
Example 3:
Find the values of x and y in the parallelogram below.



$x =$ ________

$y =$ ________

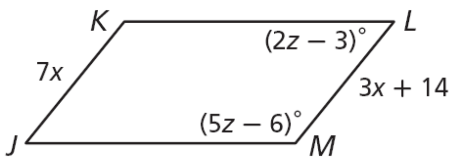
Example 4:
Find the values of x and y in the parallelogram below.



$x =$ ________

$y =$ ________

Your Turn:
JKLM is a parallelogram. Find each indicated measure.

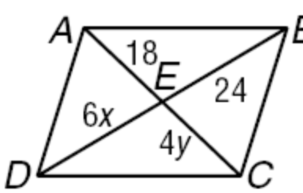


$JK =$ ________ $LM =$ ________

$m\angle L =$ ________ $m\angle M =$ ________

$m\angle J =$ ________ $m\angle K =$ ________

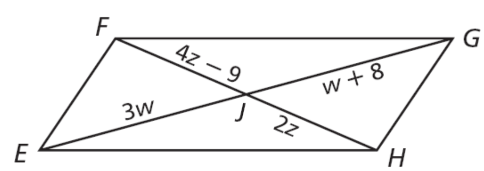
Example 5:
Find the values of x and y in the parallelogram below.



$x =$ ______

$y =$ ______

Your Turn:
EFGH is a parallelogram. Find each indicated measure.



$JG =$ ________ $JE =$ ________

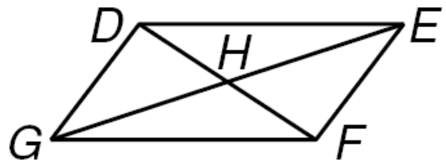
$FJ =$ ________ $HF =$ ________

$EG =$ ________ $JH =$ ________

Instruction

Example 6:

Complete each statement about parallelogram *DEFG*. Justify your answer.



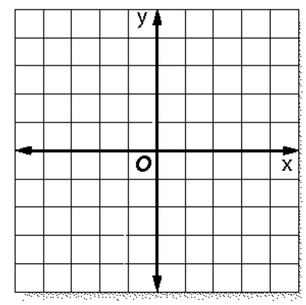
Statement	Justification
1. $\overline{DG} \parallel$ _______	1. ______________________________
2. $\overline{DE} \cong$ _______	2. ______________________________
3. $\overline{GH} \cong$ _______	3. ______________________________
4. $\angle DEF \cong$ ________	4. ______________________________
5. $\angle EFG$ is supplementary to ________ and _________	5. ______________________________

6.2 Proving a Quadrilateral is a Parallelogram

Targets

- I can recognize the conditions that ensure a quadrilateral is a parallelogram.
- I can prove that a set of points forms a parallelogram in the coordinate plane.

Instruction

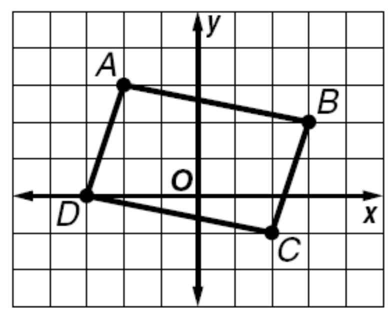
Term/Concept	Definition/Example	Picture
Parallel Segments	Two segments are **parallel** if they have the same _________.	

Example 1:
Quadrilateral *ABCD* has vertices *A*(-2, 3) *B*(3, 2) *C*(2, -1) *D*(-3, 0).

Show that quadrilateral *ABCD* is a parallelogram. Justify your answer.
Use the **slope formula.**

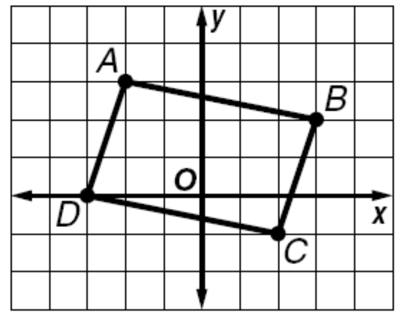
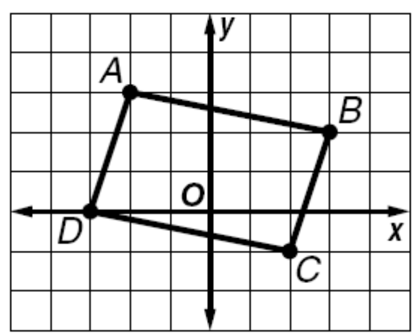
For 2 points (x_1, y_1) and (x_2, y_2) $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

(*You must show that both pairs of opposite sides are parallel – that is, you must show that opposite sides have the same slope.*)



ABCD is a parallelogram.

Justification ____

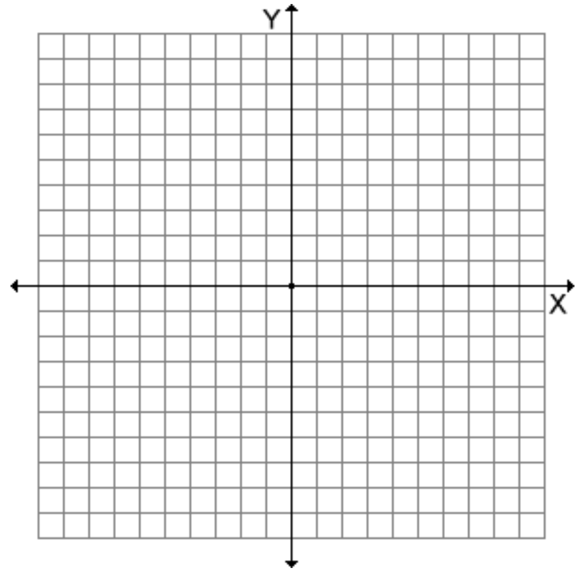
Instruction	***Example 2:*** Show that quadrilateral *ABCD* is a parallelogram. Justify your answer. Use the **distance formula**. *Distance between 2 points* (x_1, y_1) *and* (x_2, y_2) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (*You must show that both pairs of opposite sides are congruent – that is, you must show that opposite sides have the same length.*) $A(-2, 3)$ $B(3, 2)$ $C(2, -1)$ $D(-3, 0)$  *ABCD* is a parallelogram. Justification ____
	Example 3: Show that quadrilateral *ABCD* is a parallelogram. Justify your answer. Use the **midpoint formula**. For 2 points (x_1, y_1) and (x_2, y_2) midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (*You must show that both diagonals bisect each other which can be demonstrated by showing that the midpoint of both diagonals is the same.*) $A(-2, 3)$ $B(3, 2)$ $C(2, -1)$ $D(-3, 0)$  *ABCD* is a parallelogram. Justification ____

Instruction

Example 4:

Determine whether a figure with the following vertices is a parallelogram using any method. Justify your answer by showing all calculations.

Q(-3, -6), *R*(2, 2), *S*(-1, 6), *T*(-5, 2)



QRST   is / is not   a parallelogram.

Justification ____

Summary: Methods for Proving that a Quadrilateral is a Parallelogram:

1. Show that the opposite sides are ________________ using _________________.

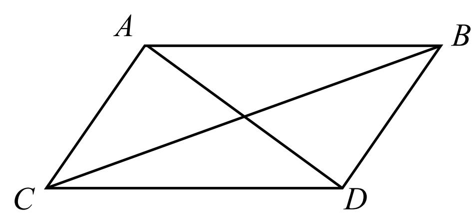
   (Show ______________ and _____________)

2. Show that the opposite sides are ________________ using _________________.

   (Show ______________ and _____________)

3. Show that the diagonal ______________ using _____________________.

   (Show ______________ and _____________)



6.3 Properties of Rectangles

Targets

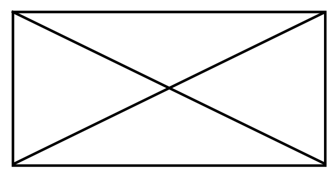
- I can recognize and apply properties of rectangles.
- I can determine whether parallelograms are rectangles.

Instruction

Term/Concept	Definition/Example	Picture
Definition of a Rectangle	A **rectangle** is a ___________________ with _________________________.	

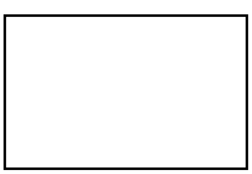
Properties of rectangles:

- Opposite sides are ___________________________
- Opposite sides are ___________________________
- All four angles are ___________________________
- Opposite angles are ___________________________
- Consecutive angles are ________________________
- Diagonals __________________________________
- Diagonals __________________________________

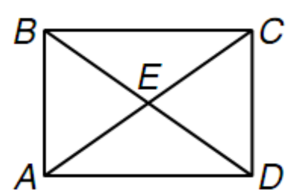


All rectangles are parallelograms, so all of the properties of parallelograms apply to rectangles.

Example 1:
In rectangle $RSTU$, $US = 6x + 3$ and $RT = 7x - 2$. Find x.

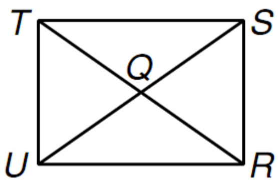


Example 2:
$ABCD$ is a rectangle. If $BE = 6y + 2$ and $CE = 4y + 6$, find y.



Instruction

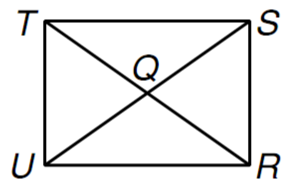
Your Turn:
In rectangle *TSRU*, $TQ = 6x + 3$ and $UQ = 7x - 2$. Find the indicated measures.



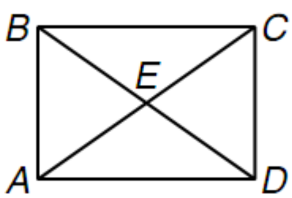
$SU =$ __________

$QR =$ __________

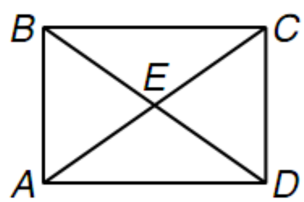
Example 3:
In rectangle *RSTU*, $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$. Find $m\angle STR$.



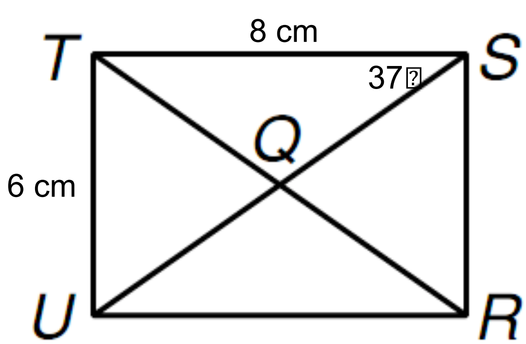
Example 4:
In rectangle *ABCD*, $m\angle BCA = 10x + 7$ and $m\angle DAC = 6x + 23$. Find $m\angle ACD$.



Your Turn:
In rectangle *ABCD*, $m\angle CBD = 3x + 6$ and $m\angle DBA = 5x - 4$. Find $m\angle ADB$.



Example 5:
Find the indicated lengths and angle measures in rectangle *TSRU* below.



$RU =$ ______ $\quad SR =$ _________

$TR =$ ______ $\quad QU =$ ________

$m\angle RSU =$ _______ $\quad m\angle SUR =$ __________

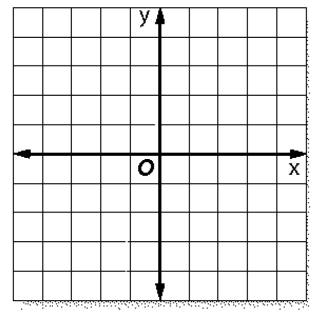
$m\angle URT =$ _______ $\quad m\angle UQR =$ _________

6.4 Proving a Quadrilateral is a Rectangle

Targets

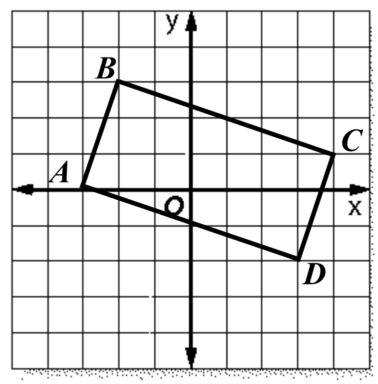
- I can recognize the conditions that ensure a quadrilateral is a rectangle.
- I can prove that a set of points forms a rectangle in the coordinate plane.

Instruction

Term/Concept	Definition/Example	Picture
Perpendicular Segments	Two segments are **perpendicular** if their ______ are ______________________.	

Example 1:
Verify that *A*(-3, 0), *B*(-2, 3), *C*(4, 1), and *D*(3, -2) are vertices of a rectangle. Justify your answer.

(*You must show that all 4 angles are right angles. This can be demonstrated by showing that consecutive sides are perpendicular*).



ABCD is a rectangle.

Justification: ____

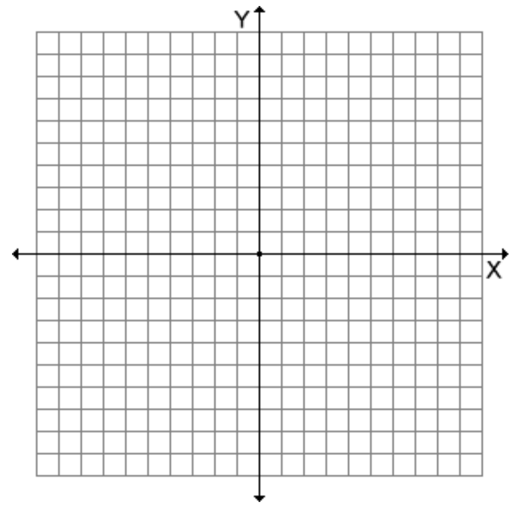
Instructi

Example 2:
Verify that *A*(-3, 0), *B*(-2, 3), *C*(4, 1), and *D*(3, -2) are vertices of a rectangle. Justify your answer.

Step 1: Show that the diagonals bisect each other (midpoint formula)
Step 2: Show that the diagonals are congruent (distance formula)

Instruction

Example 3:
Determine whether $B(0, 5)$, $G(4, 7)$, $H(5, 4)$, and $L(1, 2)$ are vertices of a rectangle using any method. Justify your answer by showing all calculations.



BGHL is / is not a rectangle.

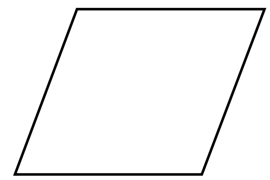
Justification: ____.

6.5 Properties of Rhombi and Squares

Targets

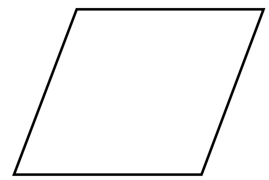
- I can recognize and apply the properties of rhombi.
- I can recognize and apply the properties of squares.

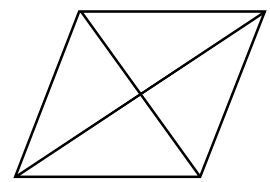
Instruction

Term/Concept	Definition/Example	Picture
Definition of a Rhombus	A **rhombus** is a ____________________ with ______________________________.	

Properties of rhombi:

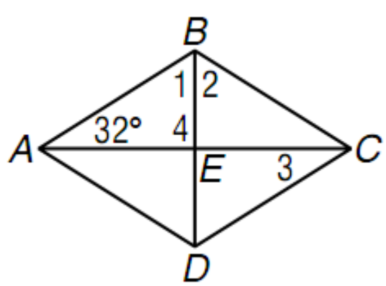
- Opposite sides are ______________________________
- Opposite sides are ______________________________
- All four sides are ______________________________
- Opposite angles are _____________________________
- Consecutive angles are ___________________________
- Diagonals ______________________________________
- Diagonals ______________________________________





All rhombi are parallelograms, so all of the properties of parallelograms apply to rhombi.

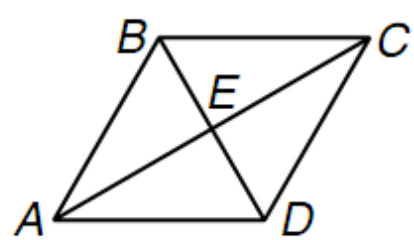
Example 1:
In rhombus $ABCD$, $m\angle BAC = 32^{\circ}$. Find the measure of each numbered angle.



$m\angle 1 =$ _______ $m\angle 2 =$ _______

$m\angle 3 =$ _______ $m\angle 4 =$ _______

Your turn:
$ABCD$ is a rhombus. Suppose $m\angle ABD = 60^{\circ}$. Find the measure of each angle.

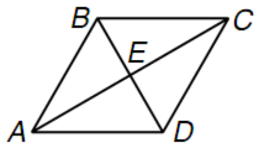


$m\angle AED =$ _______ $m\angle BDC =$ _______

$m\angle ABC =$ _______ $m\angle BCE =$ _______

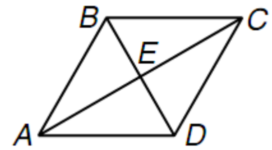
$m\angle DCE =$ _______ $m\angle DAB =$ _______

Example 2:
ABCD is a rhombus. If *AB* = 26, *BD* = 20, and *EC* = 24, find the following lengths.

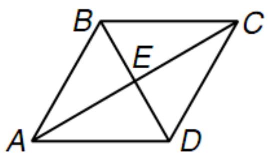


BC = _____ *BE* = _____ *AE* = ___

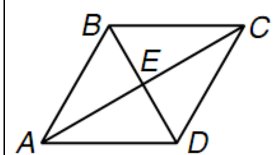
Your turn:
ABCD is a rhombus. If *AD* = 2*x* + 4 and *CD* = 4*x* – 4, find *x*. Then find the perimeter of *ABCD*.



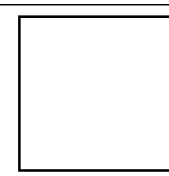
Example 3:
ABCD is a rhombus. If *AE* = 3*y* – 1 and *AC* = 16, find *y*.



Your turn:
ABCD is a rhombus. If *BE* = 5 and *AC* = 24, find the perimeter of *ABCD*.

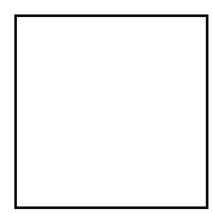


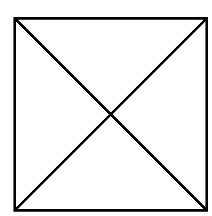
Instruction

Term/Concept	Definition/Example	Picture
Definition of a Square	A **square** is a __________________ that is both a ______________ and a ____________________.	

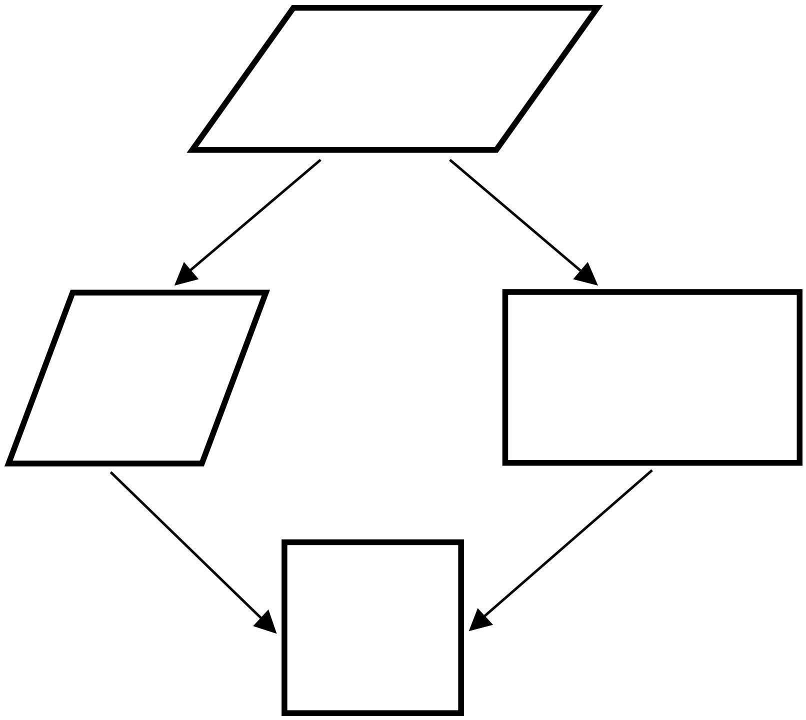
Properties of squares:

- Opposite sides are ___________________________
- Opposite sides are ___________________________
- All four sides are ___________________________
- Opposite angles are ___________________________
- All four angles are ___________________________
- Consecutive angles are ________________________
- Diagonals __________________________________
- Diagonals __________________________________
- Diagonals __________________________________

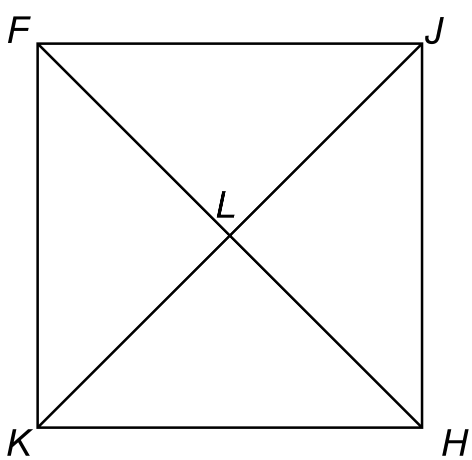




Quadrilateral Hierarchy (partial)



Example 4:
KFJH is a square with perimeter of 64 inches. Find the indicated lengths and angle measures.



$m\angle FJH$ $m\angle JHF=$ _________

$m\angle KFL$ $m\angle KLF$

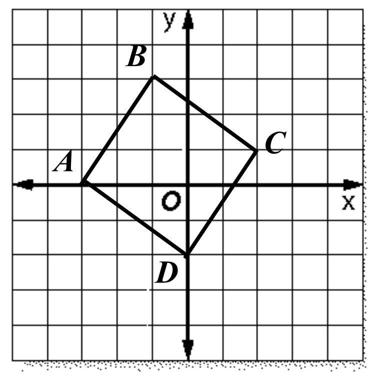
6.6 Proving that a Quadrilateral is a Rhombus or a Square

Targets

- I can recognize the conditions that ensure a quadrilateral is a rhombus
- I can recognize the conditions that ensure a quadrilateral is a square
- I can prove that a set of points forms a rhombus or square in the coordinate plane.

Instruction

Example 1:
Determine whether the given vertices form a **parallelogram**, **rectangle**, **rhombus**, or **square**. Choose all that apply. Justify your reasoning by showing all calculations.
A(-3, 0), *B*(-1, 3), *C*(2, 1), *D*(0, -2)

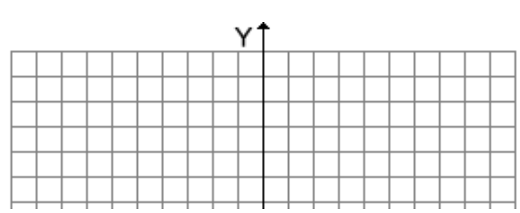


ABCD is a (circle all that apply)

Parallelogram *Rectangle* *Rhombus* *Square*

Instruction

Example 2:
Determine whether the given vertices form a **parallelogram**, **rectangle**, **rhombus**, or **square**. Choose all that apply. Justify your reasoning by showing all your calculations.
Q(-6, -1), *R*(4, -6), *S*(2, 5), *T*(-8, 10)

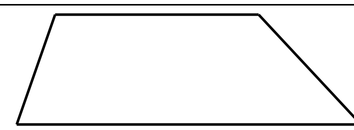
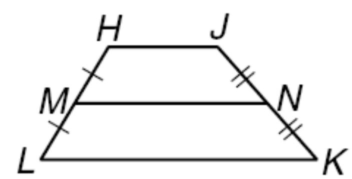


6.7 Trapezoids

Targets

- I can recognize and apply the properties of trapezoids.
- I can solve problems involving the medians of trapezoids.

Instruction

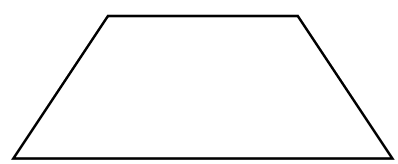
Term/Concept	Definition/Example	Picture
Definition of a Trapezoid	A **trapezoid** is a **quadrilateral** with at least **one pair** of **parallel sides**.	
Definition of an Isosceles Trapezoid	An **isosceles trapezoid** is a **trapezoid** with **congruent legs**.	
Median of a Trapezoid	The **median** of a **trapezoid** is the segment that joins the midpoints of the legs of the trapezoid.	
Theorem	The **median** of a **trapezoid** is parallel to the bases and its measure is ________ the ________ of the measures of the __________.	

Properties of trapezoids:

- Opposite sides (bases) are ___________________________
- Consecutive angles between the bases are ____________________

Properties of isosceles trapezoids:

- All of the properties above, plus
- Base angles are ____________________________
- Opposite angles are _____________________________
- Opposite sides (legs) are __________________________
- Diagonals _____________________________________



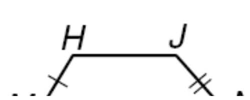
All parallelograms are trapezoids. Are trapezoids parallelograms? Yes/No

Instructi

Example 1:

$\overline{MN}$ is the median of trapezoid *HJKL*. Find each indicated value.

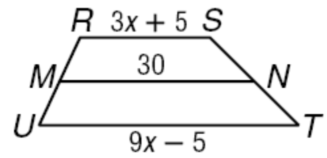
a. Find *MN* if *HJ* = 32 and *LK* = 60



Instruction

Example 2:

$\overline{MN}$ is the median of trapezoid *URST*. Find the value of x.

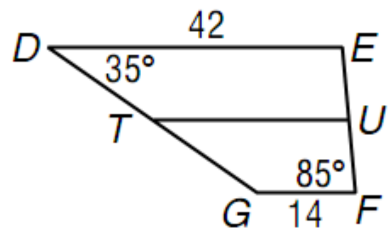


Example 3:

For trapezoid *DEFG*, *T* and *U* are the midpoints of the legs.

a. Find $m\angle E$.

b. Find $m\angle G$.



Your Turn:

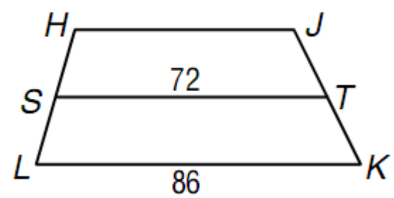
For isosceles trapezoid *HJKL*, *S* and *T* are the midpoints of the legs, and $m\angle K = 70°$.

a. Find *HJ*.

b. Find $m\angle L$.

c. Find $m\angle H$.

d. Find $m\angle J$.



Instruction

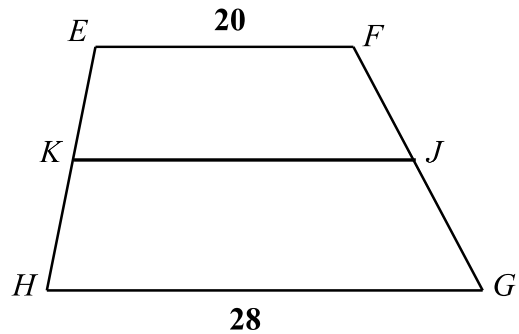
Example 4:

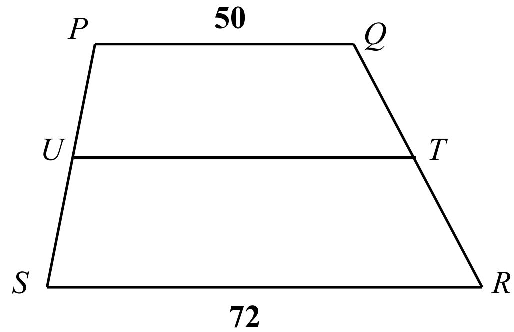
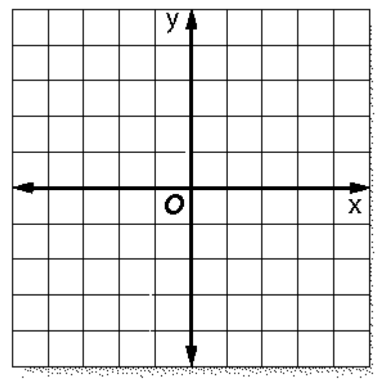
In trapezoid *EFGH*, *J* and *K* are midpoints of the legs. Let $\overline{XY}$ be the median of *EFJK*.

a. Draw and label $\overline{XY}$ on the figure.

b. Find *XY*.

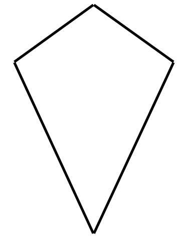
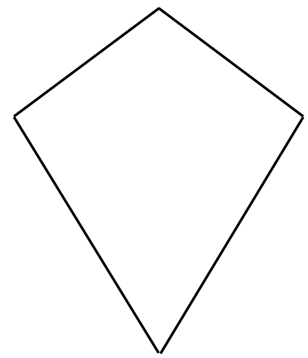
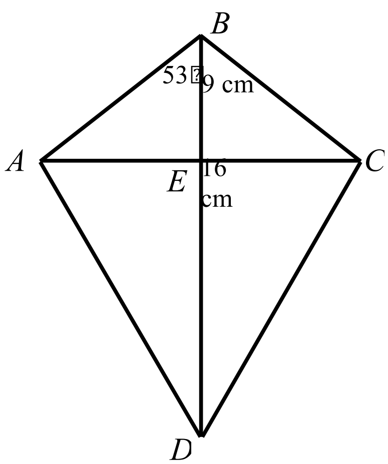
c. Suppose $m\angle H = 82°$. Name all the other angles that measure 82°.



	Your Turn: In trapezoid *PQRS*, *T* and *U* are midpoints of the legs. Let $\overline{VW}$ be the median of *UTRS*.  a. Draw and label $\overline{VW}$ on the figure. b. Find *VW*. c. Suppose $m\angle Q = 97°$. Find $m\angle UTQ$.
	Example 5: Verify that *A*(2, 4), *B*(4, 0), *C*(-2, -3), and *D*(-2, 2), are vertices of a trapezoid. Justify your answer.  *ABCD* is a trapezoid. Justification: ____

6.8 Kites and the Quadrilateral Hierarchy

Targets	○ I can recognize and apply the properties of kites. ○ I can use the quadrilateral hierarchy		
I	**Term/Concept**	**Definition/Example**	**Picture**

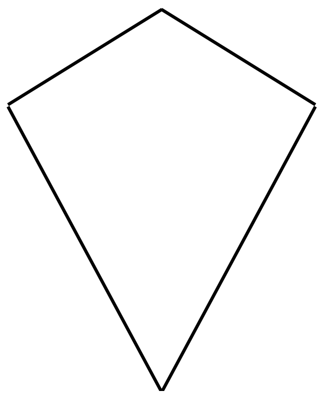
<table>
<tr><td></td><td>Definition of a Kite</td><td>A <u>kite</u> is a ____________________ with two pairs of __________________ sides that are congruent.</td><td></td></tr>
<tr><td></td><td colspan="3">Properties of kites:
• Consecutive sides are _________________________
• Diagonals are ________________________________
• Opposite angles not at the ends of the kite are ____________
• The diagonal that intersects the ends of the kite
_______________________the other diagonal
</td></tr>
<tr><td></td><td colspan="3">Example 1:
ABCD is a kite with ends B and D. If AC = 24 cm, find the indicated lengths and angle measures.


$m\angle AEB$ $m\angle EAB =$ _________</td></tr>
</table>

Instruction

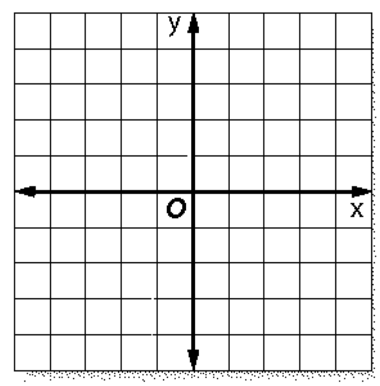
Example 2

Given ABCD is a kite with ends *B* and *D*, solve for *x* and find all missing side lengths.



Example 3:

Verify that *A*(-3, 1), *B*(-2, 4), *C*(1, 3), and *D*(1, -2), are vertices of a kite. Justify your answer.

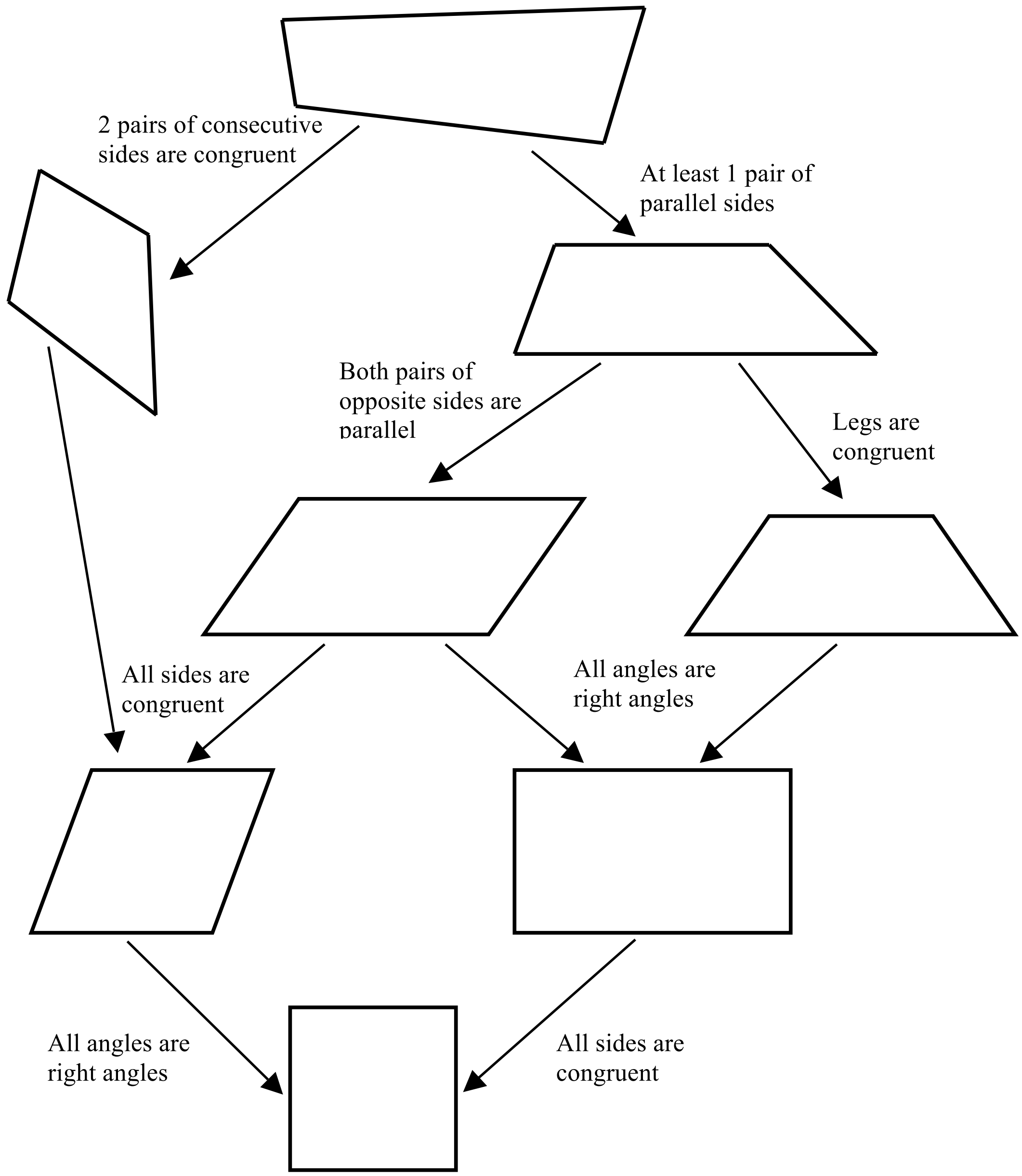


ABCD is a kite.

Justification: ____

Example 4: Complete the quadrilateral hierarchy:

QUADRILATERAL HIERACHY



Your turn: True or false?

a. A square is always a parallelogram.

b. A parallelogram is always a rectangle.

c. The diagonals of a rhombus are always congruent.

d. A trapezoid always has two congruent angles.

e. In a kite, the diagonals are always perpendicular.

6.9 Constructions of Quadrilaterals

Targets	o I can construct a parallelogram. o I can construct a rectangle o I can construct a rhombus o I can construct a square

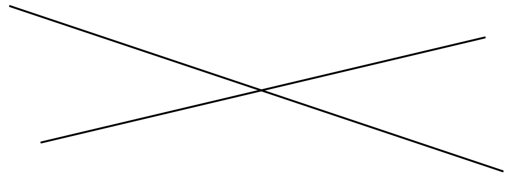
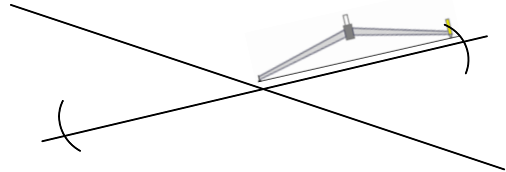
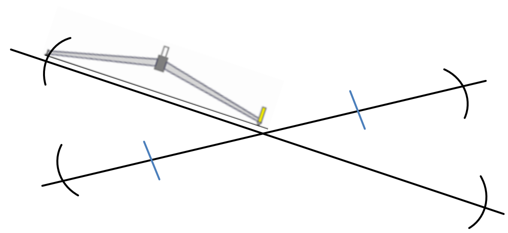
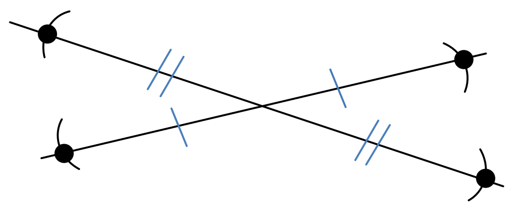
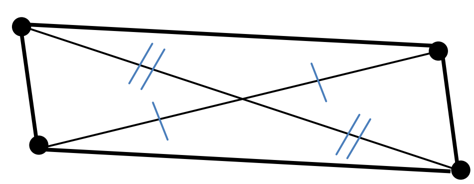
Example 1: Construct a parallelogram (diagonals ________________)

Example 2: Construct a rectangle (diagonals ________________ and ______________)

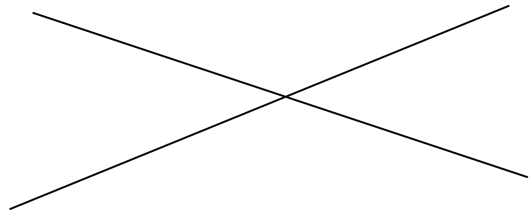
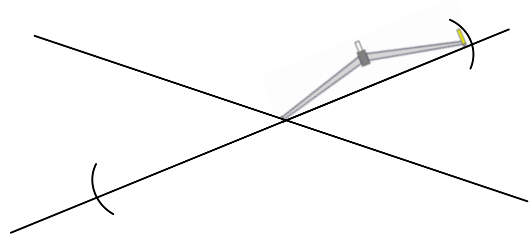
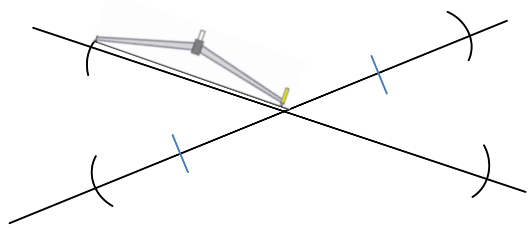
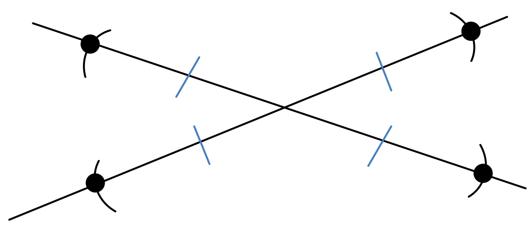
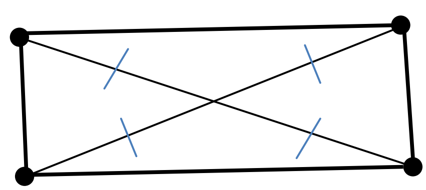
Example 3: Construct a rhombus (diagonals ________________ and _______________)

Example 4: Construct a square
(diagonals ______________ , _______________ and _______________)

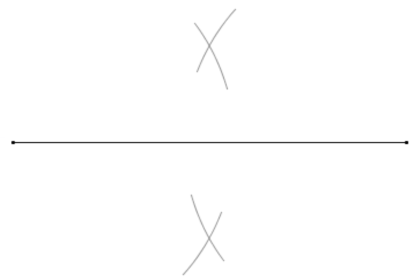
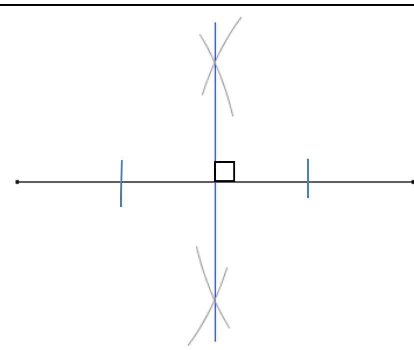
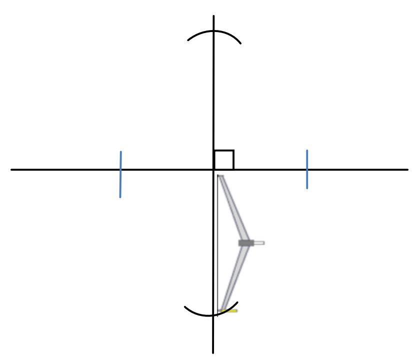
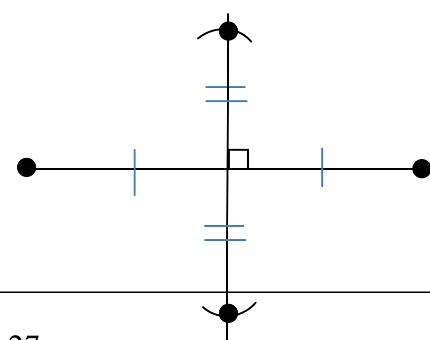
Example 1: **CONSTRUCTING A PARALLELOGRAM**

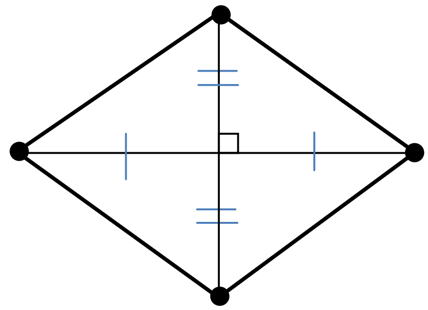
After doing this	Your work should look like this
Start by drawing two intersecting segments. These will become the diagonals of your parallelogram.	
Put the point of your compass where your segments intersect. Set the compass to a width of your choice. Mark that distance from the intersection on each side of one segment.	
Set your compas to a new width. Mark that new distance from the intersection on each side of the other segment.	
Mark the points that will be the vertices of your parallelogram.	
Connect the vertices with your straightedge.	

Example 2: **CONSTRUCTING A RECTANGLE**

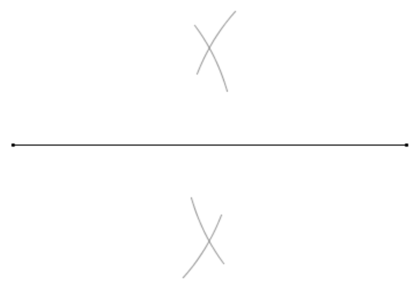
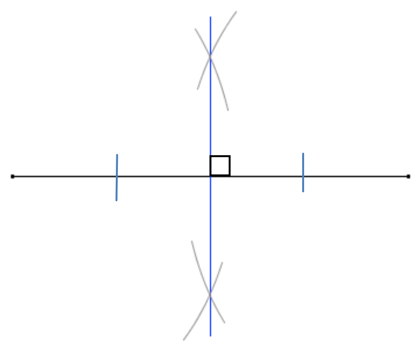
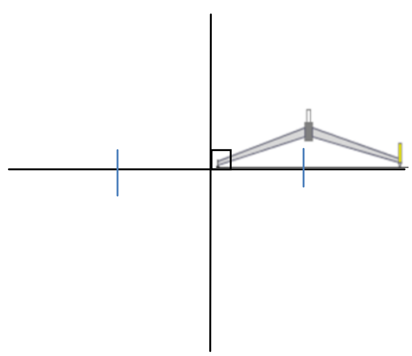
After doing this	Your work should look like this
Start by drawing two intersecting segments. These will become the diagonals of your rectangle.	
Put the point of your compass where your segments intersect. Set the compass to a width of your choice. Mark that distance from the intersection on each side of one segment.	
Keep your compass set to the same width. Mark the distance from the intersection on each side of the other segment.	
Mark the points that will be the vertices of your rectangle**.**	
Connect the vertices with your straightedge.	

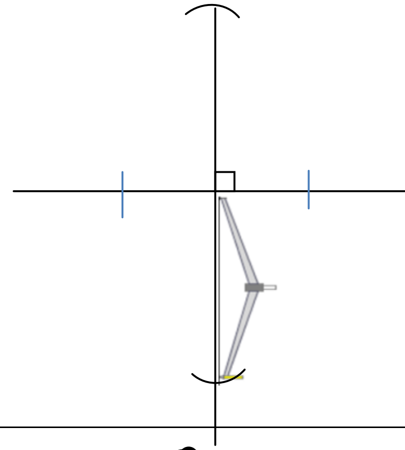
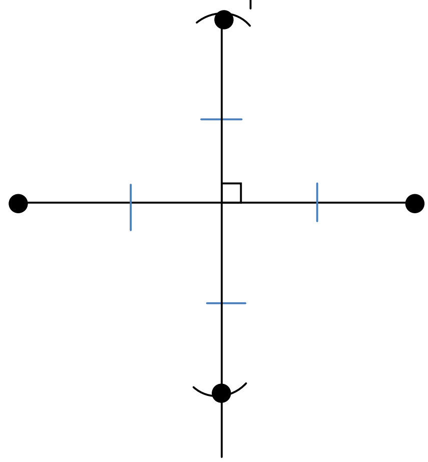
Example 3: **CONSTRUCTING A RHOMBUS**

After doing this	Your work should look like this
Start by drawing one segment. This will be one diagonal of your rhombus.	
Use your compass to construct a perpendicular bisector of your segment.	
Use your straightedge to draw in the perpendicular bisector. This is the second diagonal of your rhombus.	
Set your compass to a new width of your choice. Mark that distance from the intersection on each side of the new segment.	
Mark the points that will be the vertices of your rhombus**.**	

Connect the vertices with your straightedge.	

Example 4: **CONSTRUCTING A SQUARE**

After doing this	Your work should look like this
Start by drawing one segment. This will be one diagonal of your square.	
Use your compass to construct a perpendicular bisector of your segment.	
Use your straightedge to draw in the perpendicular bisector. This is the second diagonal of your square.	
Use your compass to measure the distance from the intersection to the end of your first diagonal.	

Mark that distance from the intersection on both sides of your other diagonal.	
Mark the points that will be the vertices of your square**.**	
Connect the vertices with your straightedge.