Lesson 7.1: Common Logarithms

Learning Targets:

- I can find common logarithms.
- I can solve logarithmic and exponential equations.
- I can use the Change of Base Formula.

What is a Common Logarithm?

Example 1: Find Common Logs with a Calculator

Use a calculator to evaluate each logarithm to four decimal places.

a) log 6

b) log 0.35

Change of Base Formula:

Example 2: Use the Change of Base Formula

Express each log in terms of common logs. Then, approximate its value to four decimal places.

a) $\log_3 16$

b) $\log_2 50$

Example 3: Use logs to solve equations where the power is the variable.

Solve the following equations. If necessary, round to four decimal places.

a) $5^x = 62$

You try: $3^x = 17$

b) $7^{2x+1} = 11$

You try: $6^{4x-3} = 8$

Lesson 7.2: Exponential Growth & Decay Story Problems

Learning Targets:

- I can solve problems involving exponential growth.
- I can solve problems involving exponential decay.

$$y = ab^x$$

a = initial amount of something
b (the *growth factor*) is written as $(1 \pm r)$
r = the growth or decay rate
x = time (as given in the problem)
$b > 1$ indicates a *growth* problem
$0 < b < 1$ indicates a *decay* problem

GROWTH	***DECAY***
$y = a(1+r)^x$	$y = a(1-r)^x$

Doubling

$$y = a(2)^x$$

Half-life

$$y = a\left(\frac{1}{2}\right)^x$$

Example 1: Doubling

An experiment begins with 300 bacteria and the population doubles every 30 minutes. How many bacteria will there be after:

a) 2 hours?

b) 10.5 hours?

Example 2: Decay

Suppose a car you bought new for $35,000 in 2008 depreciates at a rate of 18% per year.

a. Write an equation for the car's value x years after 2008.

b. What will the car's value be after 5 years?

Example 3: Growth

A computer engineer is hired for a salary of $70,400. If she gets a 5% raise each year, after how many years will she be making $100,000 or more?

Example 4: Half-life

Radium-226 has a half-life of 1,620 years.

a) Write an equation for the amount of Radium-226 remaining if there currently 550 grams after x half-life periods.

b) How much Radium-226 will remain after three half-life periods?

c) How many years are equal to three half-life periods of Radium-226?

Practice Story Problems:

1. The population of a certain strain of bacteria grows according to the formula $y = a(2)^x$, where x is the time in hours. If there are now 50 bacteria, how many will there be in 2 days?

2. The population N of a certain bacteria grows according to the equation $N = 200(2)^{1.4t}$, where t is the time in hours.

   a) How many bacteria were there at the beginning of the experiment?

   b) In how many hours will the number of bacteria reach 100,000?

3. In 2001, the population of Lagos, Nigeria was about 7,998,000. Use the population growth rate of 4.06% per year.

   a. Estimate the population in 2009.

   b. In about how many years will the population be over 50,000,000?

4. You bought a car for $28,500 in 2014. It depreciates in value at a rate of 13% each year.

   a. What is the value of the car in 2018?

   b. In how many years will the car depreciate to $5000?

5. An isotope of Cesium-137 has a half-life of 30 years.

   a. If you start with 20 mg of the substance, how many mg will be left after 90 years?

   b. After 120 years?

6. In 2010, the population of Australia was 17,800,000. In 2014, the population is now 22,000,000. At what rate is the population growing?

Lesson 7.3: Natural Logarithms

Learning Targets:

- I can understand and use base *e*.
- I can solve base *e* equations and write equivalent expressions.

<u>Base e</u>

<u>Base e and Natural Log</u>

Example 1: Write Equivalent Equations

Write an equivalent logarithmic or exponential equation for the following.

a) $e^x = 23$

b) $\ln x \approx 1.2528$

c) $e^x = 6$

d) $\ln x = 2.25$

Example 2: Evaluate Natural Logarithms

Evaluate the expressions below. (Hint: They work the same as common logs!)

a) $e^{\ln 21}$

b) $\ln e^{x^2-1}$

Example 3: Solve Equations

Evaluate the expressions below. (Hint: They work the same as common logs!)

a) $3e^{-2x} + 4 = 10$

b) $2e^{-2x} + 5 = 15$

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Example 4: Solve Pe^{rt} Problems

Suppose you deposit $700 into an account paying 6% annual interest, compounded continuously.

a) What is the balance after 8 years?

b) How long will it take for the balance in your account to reach at least $2000?

Your turn: Pe^{rt} Problem (Think-Pair-Share)

Suppose you deposit $1100 into an account paying 5.5% annual interest, compounded continuously.

a) What is the balance after 8 years?

b) How long will it take for the balance in your account to reach at least $2000?